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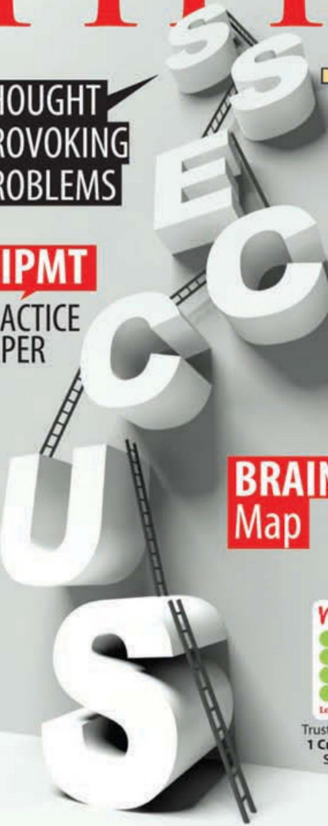

THOUGHT PROVOKING PROBLEMS

AIPMT PRACTICE PAPER

CBSE Board Sample Paper

New NCERT Xtract
Questions For Medical/Engg. Entrance Exams

BRAIN Map

PHYSICS Musing

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PHYSICS for you

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Contents

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Editor : Anil Ahlawat

editorial

To Give or Not to Give Formulae in Question Papers?

Now Hamlet's "To be or not to be" is haunting the educationists again. Before discussing whether formulae should be studied by note or understanding the subject is the question now. Formulae should never be learnt by just note. One should understand the significance of every symbol and after a step by step derivation, one arrives at the formula which can be remembered easily.

Hand-books such as the Clark's tables contain a lot of information and also the mathematical tables. They are supplied in university exams and taken back. A good student can get a lot of information but not derivation. This is just to illustrate that by giving all the formulae in question papers, the utility is marginal. Open book exams had been in vogue in many centres in India and abroad, introduced by scientists who studied abroad. The result was the same. Good students scored well with closed book or open book. The way that every formula has to be taught in the class is from experimental observations, discussions and finally the formula. The assumptions in the formula should be clearly spelt. But to learn just a formula for solving a problem is not useful.

Ultimately the purpose of the examination is to judge the effect of teaching on the student. It is not to test the ability of the students to plug in values in a given formula and get the result.

Transfer of not only technology but also the process of thinking with the maximum result till the last student has understood the topic is the secret of good teaching.

Anil Ahlawat
Editor

PHYSICS MUSING

Physics Musing was started in August 2013 issue of Physics For You with the suggestion of Shri Mahabir Singh. The aim of Physics Musing is to augment the chances of bright students preparing for JEE (Main and Advanced) / AIPMT / AIIMS / Other PMTs / PETs with additional study material.

In every issue of Physics For You, 10 challenging problems are proposed in various topics of JEE (Main and Advanced) / AIPMT. The detailed solutions of these problems will be published in next issue of Physics For You.

The readers who have solved five or more problems may send their solutions. The names of those who send atleast five correct solutions will be published in the next issue.

We hope that our readers will enrich their problem solving skills through "Physics Musing" and stand in better stead while facing the competitive exams.

By : Akhil Tewari

PROBLEM Set 7

SINGLE OPTION CORRECT

1. A smooth sphere of radius R is made to translate in a straight line with a constant acceleration $a = g$. A particle kept on the top of the sphere is released from there at zero velocity with respect to the sphere. The speed of particle with respect to the sphere as a function of angle θ as it slides down is

(a) $\sqrt{\frac{Rg(\sin\theta + \cos\theta)}{2}}$

(b) $\sqrt{Rg(1+\cos\theta - \sin\theta)}$

(c) $\sqrt{4Rg\sin\theta}$

(d) $\sqrt{2Rg(1+\sin\theta - \cos\theta)}$

2. The momentum of a body is increased by 20%. Find the percentage increase in kinetic energy (nearly).

(a) 40% (b) 22%
(c) 20% (d) 44%

3. A ball suspended by a thread swings in a vertical plane so that its acceleration values at the extreme and the mean position are equal. Find the thread's deflection angle at the extreme position.

(a) $2\tan^{-1} 2$ (b) $2\tan^{-1} 1/2$
(c) $\tan^{-1} 2$ (b) $\tan^{-1} 1/2$

4. A simple pendulum is oscillating with angular displacement 90°. For what angle with vertical is the acceleration of bob directed horizontally?

(a) $\cos^{-1}\frac{1}{3}$ (b) $\cos^{-1}\sqrt{\frac{1}{3}}$

(c) $\cos^{-1}\sqrt{\frac{1}{2}}$ (d) $\cos^{-1}\frac{1}{2}$

5. A nail is located certain distance vertically below the point of suspension of a simple pendulum. The pendulum bob is released from the position where the string makes an angle 60° from the vertical. Calculate the distance of the nail from the point of suspension such that the bob will just perform revolutions with the nail as the centre. Assume the length of the pendulum to be 1 m.

(a) 0.80 m (b) 0.60 m
(c) 0.40 m (d) 0.20 m

6. A particle of mass m slides from the top of the surface of a sphere of radius R. It looses contact and strikes the ground. At what angular position θ with upward vertical, the particle will loose contact with the surface ?

(a) $\cos^{-1}\frac{1}{3}$ (b) $\cos^{-1}\frac{2}{3}$

(c) $\cos^{-1}\frac{3}{4}$ (d) $\cos^{-1}\frac{3}{5}$

7. A small heavy block is attached to the lower end of a light rod of length l which can be rotated about its suspension point. The minimum horizontal velocity of the block so that it moves in a complete circle is

(a) $\sqrt{4gl}$ at lowest point

(b) $\sqrt{gl}$ at highest point

(c) $\sqrt{5gl}$ at lowest point

(d) zero at highest point.

8. A ball of mass m is attached to the lower end of light vertical spring of force constant k. The upper end of the spring is fixed. The ball is released from rest with the spring at its normal (unstretched) length, comes to rest again after descending through a distance x. Then

(a) $x = mg/k$

(b) $x = 2\ mg/k$

(c) The ball will have no acceleration at the position where it has descended through $x/2$

(d) The ball will have an upward acceleration equal to g at its lowermost position.

9. A man who can swim at a speed v relative to the water wants to cross a river of width d, flowing with a speed u The point opposite him across the river is P.

(a) The minimum time in which he can cross the river is $\frac{d}{v}$.

(b) He can reach the point P in time $\frac{d}{v}$.

(c) He can reach the point P in time $\frac{d}{\sqrt{v^2 - u^2}}$.

(d) He cannot reach P if $u > v$.

10. A particle moves in the xy plane with a constant acceleration 'g' in the negative y-direction. Its equation of motion is $y = ax - bx^2$, where a and b are constants. Which of the following are correct?

(a) The x-component of its velocity is constant.

(b) At the origin, the y-component of its velocity is $a\sqrt{\frac{g}{2b}}$.

(c) At the origin, its velocity makes an angle $\tan^{-1} a$ with the x-axis.

(d) The particle moves exactly like a projectile.

VERI- SIMILAR

PRACTICE PAPER 2014

Exam on 4th May

AIPMT Special

1. Turpentine oil is flowing through a tube of length l and radius r. The pressure difference between the two ends of the tube is P. The viscosity of oil is given by $\eta = \frac{P(r^2 - x^2)}{4vl}$, where v is the velocity of oil at a distance x from the axis of the tube. The dimensions of η are

(a) $[M^0L^0T^0]$ (b) $[MLT^{-1}]$
(c) $[ML^2T^{-2}]$ (d) $[ML^{-1}T^{-1}]$

2. An electric kettle has two coils. When one coil is connected to the ac mains, water in the kettle boils in 20 minutes. When the other coil is used, the same quantity of water takes 30 minutes to boil. How long will it take for the same quantity of water to boil if the two coils are connected in series?

(a) 25 minutes (b) 12 minutes
(c) 50 minutes (d) 40 minutes.

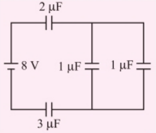
3. The efficiency of a transformer is 90%. The transformer is rated for output of 9000 W. If the primary voltage is 1000 V and resistance of primary is one ohm then the copper losses in the primary coil will be

(a) 400 W (b) 200 W
(c) 100 W (d) 300 W

4. An organ pipe closed at one end resonates with a tuning fork of frequencies 180 Hz and 300 Hz. It will also resonate with tuning fork of frequency

(a) 360 Hz (b) 420 Hz
(c) 480 Hz (d) 600 Hz

5. Figure shows four capacitors connected to an 8 V power supply. What is the potential difference across each 1 μF capacitor?



(a) 1 V (a) 2 V (c) 3 V (d) 4 V

6. Consider a uniform square plate of side a and mass M. The moment of inertia of this plate about an axis perpendicular to its plane and passing through one of its corners is

(a) $\frac{2}{3}Ma^2$ (b) $\frac{5}{6}Ma^2$
(c) $\frac{1}{12}Ma^2$ (d) $\frac{7}{12}Ma^2$

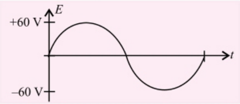
7. A smooth block released from rest on a 45° incline slides a distance d. The time taken to slide is n times as much to slide on rough incline than on a smooth incline. The coefficient of friction is

(a) $\mu_s = 1 - \frac{1}{n^2}$ (b) $\mu_s = \sqrt{1 - \frac{1}{n^2}}$
(c) $\mu_k = 1 - \frac{1}{n^2}$ (d) $\mu_k = \sqrt{1 - \frac{1}{n^2}}$

8. What is the wavelength of the most energetic photon emitted in the Balmer series of the hydrogen atom?

(a) 645 nm (b) 580 nm
(c) 435 nm (d) 365 nm

9. The equation of emf in an ac circuit is given by $E = 60\sin(100\pi t)$. What is the minimum time taken for the emf to change from +30 V to –30 V ?



(a) (1/100) s (b) 200 s
(c) 100 s (d) (1/200) s.

10. Two trains 121 m and 99 m in length are running in opposite directions with velocities 40 km h^{-1} and 32 km h^{-1}. In what time will they completely cross each other?

(a) 9 s (b) 11 s (c) 13 s (d) 15 s

11. The angle of dip at a certain place on earth is 60° and the magnitude of earth's horizontal component of magnetic field is 0.26 G. The magnetic field at the place on earth is

(a) 0.13 G (b) 0.26 G
(c) 0.52 G (d) 0.65 G

12. Two bodies of mass 10 kg and 2 kg are moving with velocities $(2\hat{i}-7\hat{j}+3\hat{k})$ m s^{-1} and $(-10\hat{i}+35\hat{j}-3\hat{k})$ m s^{-1} respectively. The velocity of their centre of mass is

(a) $2\hat{i}$ m s^{-1} (b) $2\hat{k}$ m s^{-1}
(c) $(2\hat{j}+2\hat{k})$ m s^{-1} (d) $(2\hat{i}+2\hat{j}+2\hat{k})$ m s^{-1}

13. The energy stored in a parallel plate capacitor can be treated as the energy of the electric field. The energy per unit volume, due to the electric field is

(a) E^2 (b) $\frac{1}{2}\varepsilon_0 E^2$
(c) $\frac{1}{2\varepsilon_0}E^2$ (d) $\frac{1}{2}E^2$

14. The equation of stationary wave along a stretched string is given by

$$y = 5\sin\frac{\pi x}{3}\cos 40\pi t$$

where x and y are in cm and t in second. The separation between two adjacent nodes is

(a) 1.5 cm (b) 3 cm (c) 6 cm (d) 4 cm

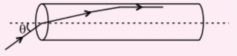
15. In the uranium radioactive series, the initial nucleus is ${}_{92}U^{238}$ and the final nucleus is ${}_{82}Pb^{206}$. When uranium nucleus decays to lead, the number of α particles and β particles emitted are

(a) 8 α, 6 β (b) 6 α, 7 β
(c) 6 α, 8 β (d) 4 α, 3 β

16. One kg of a diatomic gas is at a pressure of 8×10^4 N m^{-2}. The density of the gas is 4 kg m^{-3}. What is the energy of the gas due to its thermal motion?

(a) 3×10^4 J (b) 5×10^4 J
(c) 6×10^4 J (d) 7×10^4 J

17. A transparent solid cylindrical rod has a refractive index of $\frac{2}{\sqrt{3}}$. It is surrounded by air. A light ray is incident at the mid-point of one end of the rod as shown in the figure.



The incident angle θ for which the light ray grazes along the wall of the rod is

(a) $\sin^{-1}\left(\frac{1}{2}\right)$ (b) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$
(c) $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (d) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

18. 1 g of water at atmospheric pressure has a volume of 1 cc and when boiled it becomes 1681 cc of steam. The heat of vaporisation of water is 540 cal g^{-1}. Then the change in its internal energy in this process is

(a) 540 cal (b) 500 cal
(c) 1681 cal (d) 200 cal

19. The cylindrical tube of a spray pump has a cross-section of 8 cm^2, one end of which has 40 fine holes each of area 10^{-8} m^2. If the liquid flows inside the tube with a speed of 0.15 m min^{-1}, the speed with which the liquid is ejected through the holes is

(a) 50 m s^{-1} (b) 5 m s^{-1}
(c) 0.05 m s^{-1} (d) 0.5 m s^{-1}

20. A projectile has initially the same horizontal velocity as it would acquire if it had moved from rest with uniform acceleration of 3 m s^{-2} for 0.5 minutes. If the maximum height reached by it is 80 m, then the angle of projection is (Take g = 10 m s^{-2})

(a) $\tan^{-1}(3)$ (b) $\tan^{-1}\left(\frac{3}{2}\right)$
(c) $\tan^{-1}\left(\frac{4}{9}\right)$ (d) $\sin^{-1}\left(\frac{4}{9}\right)$

21. In a series LCR circuit the potential difference between the terminals of the inductance is 60 V, between the terminals of the capacitor is 30 V and that between the terminals of resistance is 40 V. The supply voltage will be equal to

(a) 130 V (b) 10 V
(c) 50 V (d) 70 V.

22. A galvanometer coil has resistance of 10 Ω and the metre shows full scale deflection for a current of 1 mA. The shunt resistance required to convert the galvanometer into an ammeter of range 0-100 mA is about

(a) 10 Ω (b) 1 Ω
(c) 0.1 Ω (d) 0.01 Ω.

23. A rope of mass 0.1 kg is connected at the same height of two opposite walls. It is allowed to hang under its own weight. At the contact point between the rope and the wall, the rope makes an angle θ = 10° with respect to horizontal. The tension in the rope at its midpoint between the walls is

(a) 2.78 N (b) 2.56 N
(c) 2.82 N (d) 2.71 N

24. Two thin equiconvex lenses each of focal length 0.2 m are placed coaxially with their optic centres 0.5 m apart. Then the focal length of the combination is
(a) – 0.4 m (b) 0.4 m
(c) – 0.1 m (d) 0.1 m

25. Which of the following has the longest de-Broglie wavelength if they are moving with the same velocity?
(a) Neutron (b) Proton
(c) α particle (d) β particle

26. Two radioactive substances A and B have decay constants 5λ and λ respectively. At $t = 0$ they have the same number of nuclei. The ratio of number of nuclei of A to those of B will be $(1/e)^2$ after a time interval
(a) 4λ (b) 2λ
(c) $\frac{1}{2\lambda}$ (d) $\frac{1}{4\lambda}$

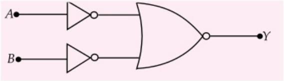
27. A long straight wire carries 10 A dc current. An electron travels perpendicular to the plane of this wire at a distance 0.1 m with velocity 5.0×10^6 m s^{-1}. Force acting on the electron due to the current in wire in newton is
(a) zero (b) 0.6×10^{-17}
(c) 1.6×10^{-17} (d) 2.2×10^{-17}

28. An ac source of frequency 50 Hz is connected in series to an inductance of 0.5 H and resistance of 157 Ω. The phase difference between current and voltage is
(a) 90° (b) 60°
(c) 30° (d) 45°

29. If λ_v, λ_x and λ_m represent the wavelengths of visible light, X-rays and microwaves respectively, then
(a) $\lambda_m > \lambda_x > \lambda_v$ (b) $\lambda_m > \lambda_v > \lambda_x$
(c) $\lambda_v > \lambda_x > \lambda_m$ (d) $\lambda_v > \lambda_m > \lambda_x$

30. What is the escape velocity for a body on the surface of a planet on which the acceleration due to gravity is $(3.1)^2$ m s^{-2} and whose radius is 8100 km?
(a) 2790 km s^{-1} (b) 27.9 km s^{-1}
(c) $\frac{27.9}{\sqrt{5}}$ km s^{-1} (d) $27.9\sqrt{5}$ km s^{-1}

31. Which logic gate is represented by the following combination of logic gates ?



(a) OR (b) NAND
(c) AND (d) NOR

32. The current gain of a transistor is 0.9. The transistor is connected in common base configuration. What would be the change in collector current when base current changes by 4 mA?
(a) 1.2 mA (b) 12 mA
(c) 24 mA (d) 36 mA

33. An organ pipe open at one end is vibrating in first overtone and is in resonance with another pipe open at both ends and vibrating in third harmonic. The ratio of length of two pipes is
(a) 3 : 8 (b) 8 : 3
(c) 1 : 2 (d) 4 : 1

34. In a series resonance LCR circuit, the voltage across R is 100 V and R = 1 kΩ, C = 2 μF. The resonant frequency is 200 rad s^{-1}. At resonance, the voltage across L is
(a) 40 V (b) 250 V
(c) 4×10^{-3} V (d) 2.5×10^{-2} V

35. A body covers 200 cm in the first 2 seconds and 220 cm in the next 4 seconds. The velocity of the body at the end of the 7th second is
(a) 10 cm s^{-1} (b) 20 cm s^{-1}
(c) 5 cm s^{-1} (d) 30 cm s^{-1}

36. The radius of a spherical nucleus as measured by electron scattering is 3.6 fm. What is the likely mass number of the nucleus?
(a) 27 (b) 40
(c) 56 (d) 120

37. A ball is dropped from a high rise platform at t = 0 starting from rest. After 6 seconds another ball is thrown downwards from the same platform with a speed v. The two balls meet at t = 18 s. What is the value of v? (Take g = 10 m s^{-2})
(a) 75 m s^{-1} (b) 55 m s^{-1}
(c) 40 m s^{-1} (d) 60 m s^{-1}

38. A conducting circular loop is placed in a uniform magnetic field, B = 0.025 T with its plane perpendicular to the loop. The radius of the loop is made to shrink at a constant rate of 1 mm s^{-1}. The induced emf when the radius is 2 cm, is
(a) 2π μV (b) π μV
(c) $\frac{\pi}{2}$ μV (d) 2 μV

39. A man in a balloon rising vertically with an acceleration of 4.9 m s^{-2} releases a stone 2 seconds after the balloon is let go from the ground. The greatest height above the ground reached by the stone is (Take $g = 9.8$ m s^{-2})
(a) 14.7 m (b) 19.6 m
(c) 9.8 m (d) 24.5 m

40. A body of mass 5 kg is moving with a momentum of 10 kg m s^{-1}. A force of 0.2 N acts on it in the direction of motion of body for 10 s. The increase in its kinetic energy is
(a) 2.8 J (b) 3.2 J (c) 3.8 J (d) 4.4 J

41. An object is tied to a string and rotated in a vertical circle of radius r. Constant speed is maintained along the trajectory. If $\frac{T_{max}}{T_{min}} = 2$, where, T_{max} = maximum tension and T_{min} = minimum tension, then $\frac{v^2}{rg}$ is
(a) 1 (b) 2 (c) 3 (d) 4

42. A block of aluminium of mass 1 kg and volume 3.6×10^{-4} m^3 is suspended from a string and then completely immersed in a container of water. The decrease in tension in the string after immersion is
(a) 9.8 N (b) 6.2 N
(c) 3.6 N (d) 1.0 N

43. An object is placed at 15 cm infront of a concave mirror whose focal length is 10 cm. The image formed will be
(a) magnified and inverted
(b) magnified and erect
(c) reduced in size and inverted
(d) reduced in size and erect.

44. A charged particle is moving in a circular orbit of radius 6 cm with a uniform speed of 3×10^6 m s^{-1} under the action of a uniform magnetic field 2×10^{-4} Wb m^{-2} at right angles to the plane of the orbit. The charge to mass ratio of the particle is
(a) 5×10^9 C kg^{-1} (b) 2.5×10^{11} C kg^{-1}
(c) 5×10^{11} C kg^{-1} (d) 5×10^{12} C kg^{-1}

45. A square loop of wire, side length 10 cm is placed at angle of 45° with a magnetic field that changes uniformly from 0.1 T to zero in 0.7 seconds. The induced current in the loop (its resistance is 1 Ω) is
(a) 1.0 mA (b) 2.5 mA
(c) 3.5 mA (d) 4.0 mA

SOLUTIONS

1. **(d):** Dimensions of $P = [ML^{-1}T^{-2}]$
Dimensions of $r = [L]$
Dimensions of $v = [LT^{-1}]$
Dimensions of $l = [L]$

$$\therefore \text{ Dimensions of } \eta = \frac{[P][(r^2 - x)^2]}{[v][l]} = \frac{[ML^{-1}T^{-2}][L^2]}{[LT^{-1}][L]} = [ML^{-1}T^{-1}]$$

2. **(c):** If V is source voltage, and R_1 and R_2 be resistances of the two coils then, if H is the heat produced in each case

$$H = \frac{V^2}{R_1}t_1, \qquad R_1 = \frac{V^2}{H}t_1$$

$$H = \frac{V^2}{R_2}t_2, \qquad R_2 = \frac{V^2 t_2}{H}$$

When resistance are put in series, then

$$H = \frac{V^2}{R_S}t_S, \qquad \therefore\ R_S = \frac{V^2}{H}t_S$$

As $R_S = R_1 + R_2$

We have $\frac{V^2}{H}t_S = \frac{V^2}{H}t_1 + \frac{V^2}{H}t_2$

or $t_s = t_1 + t_2$,
$t_s = 20 + 30 = 50$ minutes

3. **(c):** Efficiency of a transformer

$$\eta = \frac{\text{Output power}}{\text{Input power}}$$

$$\therefore \text{ Input power} = \frac{\text{Output power}}{\eta} = 9000 \times \frac{100}{90} = 10{,}000 \text{ W}$$

Also, input power $= V_P I_P$

$$\therefore \text{ Current in primary, } I_P = \frac{\text{Input power}}{V_P} = \frac{10{,}000}{1{,}000} = 10 \text{ A}$$

$\therefore$ Copper losses in the primary coil $= I_P^2 R_P = (10)^2 \times 1 = 100$ W.

4. **(b):** Resonate frequencies are $n_1\upsilon$ and $n_2\upsilon$ where n_1 and n_2 are integers and υ is fundamental frequency.
Here, $n_1\upsilon = 180$ and $n_2\upsilon = 300$

$$\because \frac{n_1}{n_2} = \frac{180}{300} = \frac{18}{30} = \frac{3}{5}$$

$\therefore$ $n_1 = 3$ and $n_2 = 5$
or $3\upsilon = 180$

$\therefore \quad \upsilon = \frac{180}{3} = 60\text{ Hz}$

Possible harmonics in the case of closed end organ pipe are 1, 3, 5, 7, 9, 11, 13, ...

The corresponding frequencies are 60 Hz, 180 Hz, 300 Hz, 420 Hz, ...

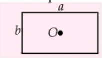
5. (c) : The total capacitance across power supply $= \frac{6}{8}\ \mu\text{F}$.

The charge on 2 μF capacitor or 3 μF capacitor $= 8 \times \frac{6}{8} = 6\ \mu\text{C}$. So the charge on each 1 μF capacitor = 3 μC.

Therefore, potential difference across each 1 μF capacitor

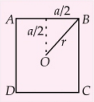
$$= \frac{\text{charge}}{\text{capacitance}} = \frac{3\ \mu\text{C}}{1\ \mu\text{F}} = 3\text{ V}$$

6. (a) : For a rectangular sheet, moment of inertia passing through O, perpendicular to the plate is



$$I_o = M\left(\frac{a^2 + b^2}{12}\right)$$

For square plate it is $\frac{Ma^2}{6}$.



$$r = \sqrt{\frac{a^2}{4} + \frac{a^2}{4}} = \frac{a}{\sqrt{2}}. \quad \therefore r^2 = \frac{a^2}{2}$$

$\therefore \quad I$ about B parallel to the axis through O is

$$I = I_o + Mr^2 = \frac{Ma^2}{6} + \frac{Ma^2}{2} = \frac{4Ma^2}{6}$$

$$I = \frac{2}{3}Ma^2$$

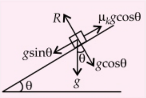
7. (c) : Component of g down the plane = $g\sin\theta$

$\therefore$ For smooth plane,

$$d = \frac{1}{2}(g\sin\theta)t^2 \qquad \text{... (i)}$$

For rough plane,

frictional retardation up the plane = μ_k ($g\cos\theta$)



$$\therefore \ d = \frac{1}{2}(g\sin\theta - \mu_k g\cos\theta)(nt)^2$$

$$\therefore \ \frac{1}{2}(g\sin\theta)t^2 = \frac{1}{2}(g\sin\theta - \mu_k g\cos\theta)n^2t^2$$

or $\quad \sin\theta = n^2 (\sin\theta - \mu_k\cos\theta)$

Putting θ = 45°, we get

or $\quad \sin45° = n^2 (\sin45° - \mu_k\cos45°)$

or $\quad \frac{1}{\sqrt{2}} = \frac{n^2}{\sqrt{2}}(1 - \mu_k)$ or $\mu_k = 1 - \frac{1}{n^2}$.

8. (d): For Balmer series

$$\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{n^2}\right] \qquad \text{where } n = 3, 4, 5 \text{....(i)}$$

By putting $n = \infty$ in equation (i), we obtain the series limit of the Balmer series

$$\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{\infty^2}\right]$$

or λ = 364.5 nm. This is the shortest wavelength in the series

Energy of photon, $E = \frac{hc}{\lambda}$

Since $E \propto \frac{1}{\lambda}$

For shortest wavelength, energy of photon is the most.

9. (a) : $E = 60\sin(100\pi t)$

$$30 = 60\sin(100\pi t_1),\ \frac{1}{2} = \sin(100\pi t_1)$$

$$100\pi t_1 = \frac{\pi}{6} \Rightarrow t_1 = \frac{1}{600}\text{ s.}$$

$$-30 = 60\sin(100\pi t_2) \Rightarrow 100\pi t_2 = \frac{7\pi}{6}$$

$$\therefore \quad t_2 = \frac{7}{600}\text{ s}$$

$$t_2 - t_1 = \frac{7}{600} - \frac{1}{600} = \frac{1}{100}\text{ s.}$$

10. (b): Here, v_A = 40 km h^{-1}, v_B = –32 km h^{-1}

Length of train A, l_A = 121 m

Length of train B, l_B = 99 m

Relative velocity of two trains is given by

$$v_{AB} = v_A - v_B = 40 - (-32)$$

$$= 72\text{ km h}^{-1} = 72 \times \frac{5}{18} = 20\text{ m s}^{-1}$$

Total distance to be travelled by each train for completely crossing the other train

$$= 121 + 99 = 220\text{ m}$$

$\therefore$ Time taken by each train to cross the other train

$$= \frac{220}{20} = 11\text{ s}$$

Hence the two trains will cross each other in 11 s.

11. (c) : Horizontal component of the earth's magnetic field, H_E = 0.26 G

Angle of dip, δ = 60°

$$\cos\delta = \frac{H_E}{B_E}$$

where B_E is the magnetic field of the earth

$\therefore\ B_E = \frac{H_E}{\cos\delta} = \frac{0.26\text{ G}}{\cos 60°} = \frac{0.26\text{ G}}{(1/2)} = 0.52\text{ G}$

12. (b): Here $m_1 = 10$ kg, $m_2 = 2$ kg

$$\vec{v}_1 = (2\hat{i} - 7\hat{j} + 3\hat{k})\text{ m s}^{-1}$$

$$\vec{v}_2 = (-10\hat{i} + 35\hat{j} - 3\hat{k})\text{ m s}^{-1}$$

$$\therefore\ \vec{v}_{\text{CM}} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$$

$$= \frac{10(2\hat{i} - 7\hat{j} + 3\hat{k}) + 2(-10\hat{i} + 35\hat{j} - 3\hat{k})}{10+2}$$

$$= 2\hat{k}\text{ m s}^{-1}$$

13. (b): Total energy $U_E = \frac{1}{2}CV^2$

$$C = \frac{\varepsilon_0 A}{d},\ V = E \times d$$

$$\therefore U_E = \frac{1}{2}\frac{\varepsilon_0 A}{d}E^2d^2 \Rightarrow U_E = \frac{1}{2}\varepsilon_0 AdE^2$$

$\therefore$ Energy per unit volume, $u_E = \frac{1}{2}\varepsilon_0\frac{E^2Ad}{Ad}$

$$\Rightarrow\quad u_E = \frac{1}{2}\varepsilon_0 E^2$$

14. (b): Given : $y = 5\sin\frac{\pi x}{3}\cos 40\pi t$

Comparing with the standard equation of stationary wave

$y = 2A\sin kx\cos\omega t$, we get

$k = \frac{\pi}{3} = \frac{2\pi}{\lambda}\ \therefore\ \lambda = 6$ cm

Hence, the separation between two adjacent nodes $= \frac{\lambda}{2} = 3$ cm

15. (a): Let number of α particles emitted be x and number of β particles emitted be y.

Difference in mass number, $4x = 238 - 206 = 32$

$\therefore\ x = 8$

Difference in charge number, $2x - 1y = 92 - 82 = 10$

or $16 - y = 10\ \therefore\ y = 6$

16. (b): The thermal energy or internal energy is

$U = \frac{5}{2}\mu RT$ for diatomic gases.

(5 is the degrees of freedom as the gas is diatomic)

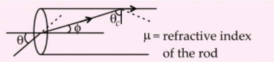
But $PV = \mu RT$

$$V = \frac{\text{mass}}{\text{density}} = \frac{1\text{ kg}}{4\text{ kg m}^{-3}} = \frac{1}{4}\text{ m}^3$$

$P = 8 \times 10^4$ N m^{-2}

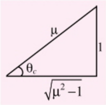
$$\therefore\ U = \frac{5}{2} \times 8 \times 10^4 \times \frac{1}{4} = 5 \times 10^4\text{ J}$$

17. (d):



If θ_c has to be the critical angle, $\theta_c = \sin^{-1}\frac{1}{\mu}$

But $\theta_c = 90° - \phi$, $\theta_i = \theta$.



$$\frac{\sin\theta_i}{\sin\phi} = \mu = \frac{2}{\sqrt{3}} \Rightarrow \frac{\sin\theta}{\cos\theta_c} = \mu.$$

But,

$$\cos\theta_c = \frac{\sqrt{\mu^2-1}}{\mu}\quad \therefore\ \sin\theta = \mu\frac{\sqrt{\mu^2-1}}{\mu} = \sqrt{\mu^2-1}.$$

$$\therefore\ \theta = \sin^{-1}\sqrt{\frac{4}{3} - 1} = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

So that θ_c is making total internal reflection.

18. (b): As the process is isobaric

$\Delta W = \int PdV = P[V_F - V_I]$

$\Delta W = 1 \times 10^6 \times [1681 - 1]$

$= 1680 \times 10^6$ erg $= 168$ J $\quad [\because 1\text{ erg} = 10^{-7}\text{ J}]$

$= 40$ cal $\quad [\because 1\text{ cal} = 4.2\text{ J}]$

$\Delta Q = mL = 1 \times 540 = 540$ cal

According to first law of thermodynamics

$\Delta Q = \Delta U + \Delta W$

$\therefore\ \Delta U = \Delta Q - \Delta W = 540 - 40 = 500$ cal.

19. (b): According to equation of continuity,

(area a) × (velocity v) = constant.

$\therefore$ For tube, $(8 \times 10^{-4}) \times \left(\frac{0.15}{60}\right) = a_1v_1$

For holes, $(40 \times 10^{-8}) \times v = a_2v_2$

$\therefore\ a_2v_2 = a_1v_1$

$$\therefore\ 40 \times 10^{-8} \times v = \frac{8 \times 10^{-4} \times 0.15}{60}$$

$$\text{or}\quad v = \frac{8 \times 10^{-4} \times 0.15}{40 \times 10^{-8} \times 60} = \frac{8 \times 15}{4 \times 6} = 5\text{ m s}^{-1}$$

20. (c): Maximum height, $H = \frac{u^2\sin^2\theta}{2g}$

or $80 = \frac{u^2\sin^2\theta}{2 \times 10}$ or $u^2\sin^2\theta = 1600$

or $u\sin\theta = 40$ m s^{-1}.

Horizontal velocity = $u \cos\theta$
As per question, $u \cos\theta = at$

$u \cos\theta = 3 \times 30 = 90 \text{ m s}^{-1}$

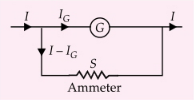
$\therefore \quad \frac{u\sin\theta}{u\cos\theta} = \frac{40}{90}$

or $\tan\theta = \frac{4}{9}$ or $\theta = \tan^{-1}\left(\frac{4}{9}\right)$.

21. (c) : In series *LCR* circuit,

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$
$$= \sqrt{(40)^2 + (60-30)^2} = \sqrt{1600+900}$$
$$= \sqrt{2500} = 50 \text{ V}.$$

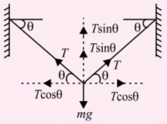
22. (c) : Here, $I_G = 1 \text{ mA} = 10^{-3}$ A
$I = 100 \text{ mA} = 100 \times 10^{-3}$ A
Galvanometer resistance, $G = 10\ \Omega$
Shunt resistance S = ?



From figure,

$$(I - I_G)S = I_G G$$
$$S = \frac{I_G G}{I - I_G} = \frac{1\times 10^{-3}\text{ A} \times 10\ \Omega}{100\times 10^{-3}\text{ A} - 1\times 10^{-3}\text{ A}}$$
$$= 0.1\ \Omega.$$

23. (c) : Mass of rope, $m = 0.1$ kg, $\theta = 10°$



From figure, $2T\sin\theta = mg$

or $T = \frac{mg}{2\sin\theta} = \frac{0.1\times 9.8}{2\sin 10°} = 2.82$ N.

24. (a) : $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$

or $\frac{1}{f} = \frac{1}{0.2} + \frac{1}{0.2} - \frac{0.5}{(0.2)(0.2)}$

or $\frac{1}{f} = 5 + 5 - 0.5 \times 5 \times 5$

or $\frac{1}{f} = 10 - 12.5 = -2.5$

or $f = -\frac{1}{2.5} = -0.4$ m

25. (d) : de Broglie wavelength

$\lambda = \frac{h}{mv}$ or $\lambda \propto \frac{1}{m}$ (when v is constant)

As $m_\alpha > m_n > m_p > m_\beta$

$\therefore \quad \lambda_\alpha < \lambda_n < \lambda_p < \lambda_\beta$

Therefore β particle has the longest de Broglie wavelength.

26. (c) : Given : $\lambda_A = 5\lambda$, $\lambda_B = \lambda$

At $t = 0$, $(N_0)_A = (N_0)_B$

After time t, $\frac{N_A}{N_B} = \left(\frac{1}{e}\right)^2$

According to radioactive decay,

$$\frac{N}{N_0} = e^{-\lambda t}$$

$$\therefore \quad \frac{N_A}{(N_0)_A} = e^{-\lambda_A t} \quad \text{... (i)}$$

$$\text{and} \quad \frac{N_B}{(N_0)_B} = e^{-\lambda_B t} \quad \text{... (ii)}$$

Divide (i) by (ii), we get

$$\frac{N_A}{N_B} = e^{-(\lambda_A - \lambda_B)t} \quad \text{or} \quad \frac{N_A}{N_B} = e^{-(5\lambda-\lambda)t}$$

or $\left(\frac{1}{e}\right)^2 = e^{-4\lambda t}$ or $\left(\frac{1}{e}\right)^2 = \left(\frac{1}{e}\right)^{4\lambda t}$

or, $4\lambda t = 2$ or $t = \frac{2}{4\lambda} = \frac{1}{2\lambda}$

27. (a) : Force on the electron, $\vec{F} = -e(\vec{v} \times \vec{B})$. As electron is travelling in a plane perpendicular to the plane of wire, so angle θ between $\vec{v}$ and $\vec{B}$ is either 0° or 180° and sin0° or sin180° = 0.

28. (d) : $\tan\phi = \frac{X_L}{R} = \frac{\omega L}{R} = \frac{2\pi\upsilon L}{R}$

where ϕ is the phase difference between current and voltage.
Substituting the values, we get

$$\tan\phi = \frac{2\pi \times 50 \times 0.5}{157} = 1$$
$$\phi = 45°$$

29. (b): $\lambda_m > \lambda_v > \lambda_x$
In electromagnetic waves spectrum X-rays has minimum wavelength and microwave has maximum wavelength.

30. (c) : Escape velocity of the body from the surface of a planet is

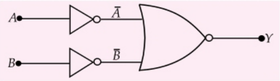
$$v_e = \sqrt{2 g_p R_p}$$

Substituting the values, we get

$$v_e = \sqrt{2\times(3.1)^2\times(8100\times10^3)}\ \text{m s}^{-1}$$

$$= \frac{3.1\times90\times10\sqrt{20}}{1000}\ \text{km s}^{-1} = \frac{27.9}{\sqrt{5}}\ \text{km s}^{-1}$$

31. (c) :



The Boolean expression of this arrangement is

$$Y = \overline{\bar{A}+\bar{B}} = \bar{\bar{A}}\cdot\bar{\bar{B}} = A\cdot B$$

Thus AND gate is produced.

32. (d) : For a transistor $\Delta I_E = \Delta I_B + \Delta I_C$

$$\alpha = \frac{\Delta I_C}{\Delta I_E} \quad \therefore \quad \alpha = \frac{\Delta I_C}{\Delta I_B + \Delta I_C}$$

Substituting the values we get

$$0.9 = \frac{\Delta I_C}{4+\Delta I_C} \quad \text{or} \quad \Delta I_C = 36\ \text{mA}$$

33. (c) : For an organ pipe open at one end,

frequency of 1st overtone $\upsilon_1 = \frac{3v}{4l_1}$

For the organ pipe open at both ends,

frequency of 3rd harmonic, $\upsilon_2 = \frac{3v}{2l_2}$

As $\upsilon_1 = \upsilon_2$

$$\therefore \quad \frac{3v}{4l_1} = \frac{3v}{2l_2} \quad \text{or} \quad \frac{l_1}{l_2} = \frac{2}{4} = \frac{1}{2}$$

34. (b) : Here, $V_R = 100$ V, $R = 1\ \text{k}\Omega = 10^3\ \Omega$, $C = 2\ \mu\text{F} = 2\times10^{-6}$ F, $\omega = 200$ rad s^{-1}

Current, $I = \frac{V}{Z}$

where $V = \sqrt{(V_R)^2 + (V_L - V_C)^2}$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

At resonance, $X_L = X_C \quad \therefore \quad Z = R$

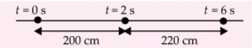
$V_L = V_C \quad \therefore \quad V = V_R$

$$\therefore \quad I = \frac{V_R}{R} = \frac{100\ \text{V}}{10^3\ \Omega} = 10^{-1}\ \text{A}$$

At resonance, $V_L = V_C = IX_C = \frac{I}{\omega C}$

$$= \frac{10^{-1}}{200\times2\times10^{-6}} = \frac{10^3}{4} = 250\ \text{V}$$

35. (a) : Let u and a be the initial velocity and uniform acceleration of the body.



Using $S = ut + \frac{1}{2}at^2$

$\therefore \quad 200 = u(2) + \frac{1}{2}a(2)^2$

$100 = u + a$ ... (i)

Also $420 = 6u + \frac{1}{2}a(6)^2$

$70 = u + 3a$ ... (ii)

Solving equations (i) and (ii), we get

$u = 115$ cm s^{-1},

$a = -15$ cm s^{-2}

Using $v = u + at$

$\therefore \quad v = 115 + (-15)7 = 10$ cm s^{-1}

36. (a) : Nuclear radius $R = R_0(A)^{1/3}$

where A is the mass number of a nucleus.

Given : $R = 3.6$ fm

$\therefore \quad 3.6\ \text{fm} = (1.2\ \text{fm})(A^{1/3}) \qquad [\because R_0 = 1.2\ \text{fm}]$

or $A = (3)^3 = 27$

37. (a) : Let the two balls meet after t s at distance x from the platform.

For the first ball

$u = 0$, $t = 18$ s, $g = 10$ m s^{-2}

Using $h = ut + \frac{1}{2}gt^2$

$\therefore \quad x = \frac{1}{2}\times10\times18^2$ ...(i)

For the second ball

$u = v$, $t = 12$ s, $g = 10$ m s^{-2}

Using $h = ut + \frac{1}{2}gt^2$

$\therefore \quad x = v\times12 + \frac{1}{2}\times10\times12^2$ ...(ii)

From equations (i) and (ii), we get

$$\frac{1}{2}\times10\times18^2 = 12v + \frac{1}{2}\times10\times(12)^2$$

or $12v = \frac{1}{2}\times10\times[(18)^2 - (12)^2]$

$$= \frac{1}{2}\times10\times[(18+12)\,(18-12)]$$

$$12v = \frac{1}{2}\times10\times30\times6$$

or $v = \frac{1\times10\times30\times6}{2\times12} = 75$ m s^{-1}

38. (b) : Here,

Magnetic field, $B = 0.025$ T

Radius of the loop, $r = 2$ cm $= 2\times10^{-2}$ m

Constant rate at which radius of the loop shrinks,

$$\frac{dr}{dt} = 1\times10^{-3}\ \text{m s}^{-1}$$

Magnetic flux linked with the loop is

$$\phi = BA\cos\theta = B(\pi r^2)\cos0^\circ = B\pi r^2$$

The magnitude of the induced emf is

$$|\varepsilon| = \frac{d\phi}{dt} = \frac{d}{dt}(B\pi r^2) = B\pi 2r\frac{dr}{dt}$$

$$= 0.025 \times \pi \times 2 \times 2 \times 10^{-2} \times 1 \times 10^{-3}$$

$$= \pi \times 10^{-6}\ \text{V} = \pi\ \mu\text{V}$$

39. (a) : Here, $a = 4.9$ m s^{-2}, $t = 2$ s, $u = 0$,

$$s = ut + \frac{1}{2}at^2$$

$$s = 0 + \frac{1}{2} \times 4.9 \times (2)^2 = 9.8\ \text{m}$$

This is the height from where stone is dropped

Upward velocity of stone when released,

$v = u + at = 0 + 4.9 \times 2 = 9.8$ m s^{-1}

The stone will move up till its velocity become zero.

From, $v^2 - u^2 = 2as$

$$0 - (9.8)^2 = 2(-9.8)s' \quad \therefore \quad s' = 4.9\ \text{m}$$

Maximum height above the ground

$$= s + s' = 9.8\ \text{m} + 4.9\ \text{m} = 14.7\ \text{m}$$

40. (d) : Initial kinetic energy $= \frac{p_1^2}{2m} = \frac{10 \times 10}{2 \times 5} = 10$ J

Impulse $= p_2 - p_1 = F \times t$

$p_2 - 10 = 0.2 \times 10$ or $p_2 = 12$ kg m s^{-1}

Final kinetic energy $= \frac{p_2^2}{2m} = \frac{12 \times 12}{2 \times 5} = 14.4$ J

Increase in kinetic energy = 14.4 J – 10 J = 4.4 J

41. (c) : At the lowest point, $\frac{mv^2}{r} = T_L - mg$ ...(i)

At the highest point, $\frac{mv^2}{r} = T_H + mg$ ...(ii)

As $\frac{T_{\max}}{T_{\min}} = \frac{T_L}{T_H} = 2 \quad \therefore \quad T_L = 2T_H$

As according to question, constant speed is maintained, therefore from equations (i) and (ii), we get

$T_L - mg = T_H + mg$

$2T_H - mg = T_H + mg$

$T_H = 2mg$

From equation (ii), $\frac{mv^2}{r} = 3mg$ or $\frac{v^2}{rg} = 3$

42. (c) : Here, mass of block, $m = 1$ kg

Volume of a block, $V = 3.6 \times 10^{-4}$ m^3

Tension in the string, $T = mg$

When a block is immersed completely in water, its weight becomes

$mg' = mg$ – Upthrust

$= mg - V\rho_{\text{water}}g$

$\therefore$ Tension in the string, $T' = mg'$

$= mg - V\rho_{\text{water}}g$

$\therefore$ Decrease in the tension of the string $= T - T'$

$= mg - [mg - V\rho_{\text{water}}g] = V\rho_{\text{water}}g$

$= (3.6 \times 10^{-4}\ \text{m}^3) \times (10^3 \times \text{kg m}^{-3}) \times (10\ \text{m s}^{-2})$

$= 3.6$ N

43. (a) : According to Cartesian sign convention

Object distance, $u = -15$ cm

Focal length, $f = -10$ cm

Using mirror formula

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \quad = \frac{1}{(-15)} + \frac{1}{v} = \frac{1}{(-10)}$$

$$\frac{1}{v} = \frac{1}{(-10)} - \frac{1}{(-15)} = \frac{1}{(-10)} + \frac{1}{(15)}$$

or $v = -30$ cm

The image is 30 cm from the mirror on the same side of the object.

Magnification, $m = -\frac{v}{u} = -\frac{(-30\ \text{cm})}{(-15\ \text{cm})} = -2$

The image is magnified, real and inverted.

44. (b) : Here, $v = 3 \times 10^6$ m s^{-1}

$B = 2 \times 10^{-4}$ Wb m^{-2} $= 2 \times 10^{-4}$ T

$R = 6$ cm $= 6 \times 10^{-2}$ m

As $Bqv = \frac{mv^2}{R}$ or $\frac{q}{m} = \frac{v}{BR}$

Substituting the given values, we get

$$\frac{q}{m} = \frac{3 \times 10^6}{2 \times 10^{-4} \times 6 \times 10^{-2}} = 0.25 \times 10^{12}\ \text{C kg}^{-1}$$

$$= 2.5 \times 10^{11}\ \text{C kg}^{-1}$$

45. (a) : Area of a square loop, $A = 10$ cm $\times$ 10 cm

$A = 100$ cm^2 $= 100 \times 10^{-4}$ m^2 $= 10^{-2}$ m^2

Initial magnetic flux linked with the loop

$\phi_1 = B_1 A\cos\phi = 0.1 \times 10^{-2} \times \cos 45°$

$$= \frac{0.1 \times 10^{-2} \times 1}{\sqrt{2}} = \frac{10^{-3}}{\sqrt{2}}\ \text{Wb}$$

Final magnetic flux linked with loop

$\phi_2 = 0$ Wb $\quad (\because\ B_2 = 0)$

The induced emf in the loop is

$$\varepsilon = -\frac{d\phi}{dt} = -\frac{(\phi_2 - \phi_1)}{t}$$

$$= -\frac{\left(0 - \frac{10^{-3}}{\sqrt{2}}\right)}{0.7} = \frac{10^{-3}}{0.7 \times \sqrt{2}} = 10^{-3}\ \text{V}$$

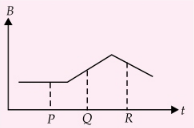
The induced current in the loop is

$$I = \frac{\varepsilon}{R} = \frac{10^{-3}\ \text{V}}{1\ \Omega} = 10^{-3}\ \text{A} = 1.0\ \text{mA}$$

TEST YOUR SKILLS

1. A boy moves along x-axis. What is the sign of his acceleration in the following cases?
   (a) If he is moving in the positive direction with increasing speed
   (b) If he is moving in the positive direction with decreasing speed
   (c) If he is moving in the negative direction with increasing speed
   (d) If he is moving in the negative direction with decreasing speed.

2. Given figure shows a graph of the magnitude of B versus time t for a magnetic field that passes through a fixed loop and is oriented perpendicular to the plane of the loop. The magnitude of the emf produced in the loop at three equal instants indicated P, Q and R. Mark the point at which smallest to largest emf.



   (a) $R > Q > P$ (b) $Q > R > P$
   (c) $P > Q > R$ (d) $Q = R > P$

3. A large truck and a small car collide and stick together. Which one undergoes the larger change in momentum?
   (a) The car. (b) The truck.
   (c) The momentum change is the same for both vehicles.
   (d) You can't tell, without knowing the final velocity of the combined masses.

4. A mass m undergoes circular motion close to the earth's surface. The speed of the mass is constant. Considering only the information given and making no further assumptions about the system, which of the following statements is definitely true?
   (a) The acceleration of the mass is zero because its speed is constant.
   (b) The force on the mass is directed radially outward away from the centre of circular motion.
   (c) The force on the mass is directed radially outwards. The acceleration is radially inwards towards the centre of circular motion.
   (d) Both the net force and net acceleration are radially inwards.

5. An object hangs motionless from a spring. When the object is pulled down, the sum of the elastic potential energy of the spring and the gravitational potential energy of the object and Earth
   (a) increases (b) decreases
   (c) stays the same
   (d) can either increase or decrease depending on the spring constant.

6. A person at a distance R from the centre of the earth (where R is greater than the radius of the earth) is attracted towards the earth by a gravitational force of 400 Newton. How far away from the centre of the earth must the person be for the gravitational force to be 100 Newton?
   (a) $\frac{1}{4}R$ (b) $\frac{1}{2}R$ (c) $2R$ (d) $4R$

7. Colin claims to have invented a machine that removes heat energy from the air and converts it into electricity. The entire machine is at the same temperature as the surrounding air and does not have an external power supply. Can Colin's machine work?
   (a) Yes, it can.
   (b) No, it cannot, because there is no electricity in the air.

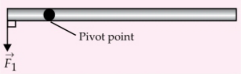
(c) No, it cannot, because there will be no net flow of energy from the air into a machine at the same temperature.

(d) No, it cannot, because it would violate conservation of energy.

8. The figure shows an overhead view of a meter stick that can pivot about the point indicated, which is to the left of the stick's midpoint. Two horizontal forces, $\vec{F}_1$ and $\vec{F}_2$, are applied to the stick. Only $\vec{F}_1$ is shown. Force $\vec{F}_2$ is perpendicular to the stick and is applied at the right end. If the stick is not to turn,

(a) what should be the direction of $\vec{F}_2$, and

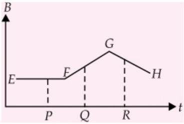
(b) should F_2 be greater than, less than, or equal to F_1?



SOLUTIONS

1. If the signs of the velocity and acceleration of the boy are the same, the speed of the boy increases. If the signs are opposite, the speed decreases. Hence (a) positive, (b) negative, (c) negative and (d) positive.

2. **(b) :**



Magnitude of emf, $|\varepsilon| = \frac{d\phi}{dt} = \frac{d(BA)}{dt}$ $(\because \phi = BA)$

$= A\frac{dB}{dt}$ (As area of the loop is constant)

$\therefore \quad |\varepsilon| \propto \frac{dB}{dt}$ (Slope of graph)

Clearly $\left.\frac{dB}{dt}\right|_{FG} > \left.\frac{dB}{dt}\right|_{GH} > \left.\frac{dB}{dt}\right|_{EF}$

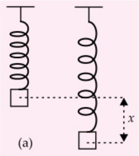
$\therefore \quad |\varepsilon|_Q > |\varepsilon|_R > |\varepsilon|_P$.

3. **(c) :** As the total momentum has to remain the same, whatever be the change in momentum of the truck will be equal and opposite to that of the car. Their magnitudes of change of momentum will be the same.

4. **(d) :**

For circular motion taking place in the inertial frame, one takes centripetal force where one assumes the force and the acceleration is always acting towards the centre.

5. **(a) :**



Potential energy of the spring is $\frac{1}{2}kx^2$. The potential energy of the spring increases with increase in x. The loss of potential energy by decrease in height is more than compensated by the spring. That is why it is pulled back.

6. **(c) :** The force due to gravity is given by $F = \frac{GMm}{R^2}$.

As G, M and m do not change, one can write an equation for the first force $F_1 = \frac{GMm}{R_1^2}$ ...(i)

and the second force $F_2 = \frac{GMm}{R_2^2}$, ...(ii)

where R_1 and R_2 are the respective distances.

Dividing (i) by (ii), we get the ratio

$$\frac{F_1}{F_2} = \left(\frac{R_2}{R_1}\right)^2 = \frac{400\text{ N}}{100\text{ N}} = 4 = 2^2.$$

Thus, $R_2 = 2R_1 = 2R$.

7. **(c) :** Energy can be converted from one form to another, hence there is no need for there to be electricity in the air, and conservation of energy need not be violated. Heat energy can be converted into electrical energy. To take energy out of the air and into the machine requires the machine to be cooler than the air, otherwise there will be no net flow of heat.

8. (a) F_2 must act downward in the figure ($\tau_{net} = 0$)
(b) F_2 should be less than F_1 to maintain the stick in balance.

NCERT Xtract

New

Questions for Medical/ Engineering Entrance Exams

Current Electricity

1. In an atom, electrons revolves around the nucleus along a path of radius 0.72 Å making 9.4×10^{18} revolution per second. The equivalent current is (Take $e = 1.6 \times 10^{-19}$ C)
(a) 1.2 A (b) 1.5 A (c) 1.4 A (d) 1.8 A

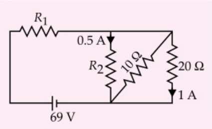
2. A battery of emf 15 V and internal resistance of 4 Ω is connected to a resistor. If the current in the circuit is 2 A and the circuit is closed. Resistance of the resistor and terminal voltage of the battery will be
(a) 2.5 Ω, 6 V (b) 3.5 Ω, 6 V
(c) 2.5 Ω, 7 V (d) 3.5 Ω, 7 V

3. Biot-Savart law indicates that the moving electrons (velocity v) produce a magnetic field B such that
(a) $B \perp v$
(b) $B \parallel v$
(c) it obeys inverse cube law
(d) it is along the line joining the electron and point of observation.

4. The resistance of the wire in the platinum resistance thermometer at ice point is 5 Ω and at steam point is 5.25 Ω. When the thermometer is inserted in an unknown hot bath its resistance is found to be 5.5 Ω. The temperature of the hot bath is
(a) 100°C (b) 200°C (c) 300°C (d) 350°C

5. An electric heater is connected to the voltage supply. After few seconds, current get its steady value then its initial current will be
(a) equal to its steady current
(b) slightly higher than its steady current
(c) slightly less than its steady current
(d) zero

6. In the circuit shown below, the resistances R_1 and R_2 are respectively

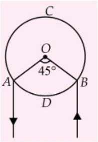


(a) 14 Ω and 40 Ω (b) 40 Ω and 14 Ω
(c) 40 Ω and 30 Ω (d) 14 Ω and 30 Ω

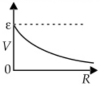
7. A cylindrical rod is reformed to half of its original length keeping volume constant. If its resistance before this change were R, then the resistance after reformation of rod will be
(a) R (b) $\frac{R}{4}$ (c) $\frac{3R}{4}$ (d) $\frac{R}{2}$

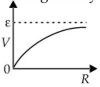
8. A current carrying wire in the shape of a circle as the current progresses along the wire the direction of current density changes in an exact manner while the current I remains unaffected. The responsible factor for it is
(a) the charges ahead.
(b) electric field produced by charges accumulated on the surface of wire.
(c) the charges just behind a given segment of wire which push them, right away by repulsion.
(d) none of these.

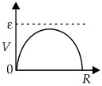
9. A and B are two points on a uniform ring of resistance 15 Ω. The $\angle AOB = 45°$. The equivalent resistance between A and B is
(a) 1.64 Ω
(b) 2.84 Ω
(c) 4.57 Ω
(d) 2.64 Ω

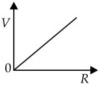


10. A cell having an emf ε and internal resistance r is connected across a variable external resistance R. As the resistance R is increased, the plot of potential difference V across R is given by

(a) 

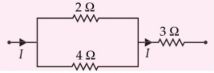
(b) 

(c) 

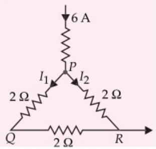
(d) 

11. When a current of 2 A flows in a battery from negative to positive terminal, the potential difference across it is 12 V. If a current of 3 A flowing in the opposite direction produces a potential difference of 15 V, the emf of the battery is
(a) 12.6 V (b) 13.2 V (c) 15.5 V (d) 14 V

12. In the circuit shown in figure, heat developed across 2 Ω, 4 Ω and 3 Ω resistances are in the ratio of
(a) 2 : 4 : 3
(b) 8 : 4 : 12
(c) 4 : 8 : 27
(d) 8 : 4 : 27



13. A current of 6 A enters one corner P of an equilateral triangle PQR having 3 wires of resistances 2 Ω each and leaves by the corner R. Then the currents I_1 and I_2 are



(a) 2 A, 4 A (b) 4 A, 2 A
(c) 1 A, 2 A (d) 2 A, 3 A

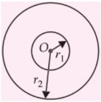
14. Three resistances 2 Ω, 4 Ω, 5 Ω are combined in series and this combination is connected to a battery of 12 V emf and negligible internal resistance. The potential drop across these resistances are respectively
(a) (5.45, 4.36, 2.18) V (b) (2.18, 5.45, 4.36) V
(c) (4.36, 2.18, 5.45) V (d) (2.18, 4.36, 5.45) V

15. A copper cylindrical tube has inner radius a and outer radius b. The resistivity is ρ. The resistance of the cylinder between the two ends is

(a) $\dfrac{\rho l}{b^2 - a^2}$ (b) $\dfrac{\rho l}{2\pi(b-a)}$

(c) $\dfrac{\rho l}{\pi(b^2 - a^2)}$ (d) $\dfrac{\pi(b^2 - a^2)}{\rho l}$

16. Space between two concentric spheres of radii r_1 and r_2, such that $r_1 < r_2$, is filled with a material of resistivity ρ. Find the resistance between inner and outer surface of the material.

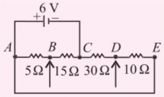


(a) $\dfrac{r_1}{r_2}\dfrac{\rho}{2}$ (b) $\dfrac{r_2 - r_1}{r_1 r_2}\dfrac{\rho}{4\pi}$

(c) $\dfrac{r_1 r_2}{r_2 - r_1}\dfrac{\rho}{4\pi}$ (d) none of these

17. If voltage across a bulb rated 220 V, 100 W drops by 2.5% of its rated value, the percentage of the rated value by which the power would decrease is
(a) 20% (b) 2.5% (c) 5% (d) 10%

18. Four resistors are connected as shown in the figure. A 6 V battery of negligible resistance is connected across terminals A and C. The potential difference across terminals B and D will be

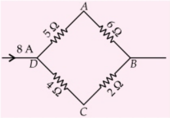


(a) zero (b) 1.5 V (c) 2 V (d) 3 V.

19. n resistors each of resistance R first combine to give maximum effective resistance and then combine to give minimum effective resistance. The ratio of the maximum to minimum resistance is
(a) n (b) n^2 (c) $n^2 - 1$ (d) n^3

20. The resistivity of alloy manganin is
(a) Nearly independent of temperature
(b) Increases rapidly with increase in temperature
(c) Decreases with increase in temperature
(d) Increases rapidly with decrease in temperature

21. A current of 8 A flows in a system of resistors as shown in figure. The potential difference $V_C - V_A$ will be

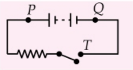


(a) $\frac{45}{23}$ V (b) 6.6 V (c) 3.3 V (d) 9.9 V

22. A potentiometer wire of length 100 cm has a resistance of 10 Ω. It is connected in series with a resistance and a cell of emf 2 V and of negligible internal resistance. A source of emf 10 mV is balanced against a length of 40 cm of the potentiometer wire. What is the value of external resistance ?
(a) 790 Ω (b) 890 Ω (c) 990 Ω (d) 1090 Ω

23. A boy has two spare light bulbs in his drawer. One is marked 240 V and 100 W and the other is marked 240 V and 60 W. Which of the following statements are correct about the light bulbs?
(a) The 60 W light bulb has more resistance and therefore burns less brightly.
(b) The 60 W light bulb has less resistance and therefore burns less brightly.
(c) The 100 W bulb has more resistance and therefore burns more brightly.
(d) The 100 W bulb has less resistance and therefore burns less brightly.

24. A battery, an open switch and a resistor are connected in series as shown below. Consider the following three statements concerning the circuit. A voltmeter will read zero if it is connected across points



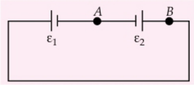
(i) P and T
(ii) P and Q
(iii) Q and T
Which one of the above is/are true?
(a) only (i) (b) only (iii)
(c) only (i) and (iii) (d) (i), (ii) and (iii)

25. In a potentiometer a cell of emf 1.5 V gives a balanced point at 32 cm length of the wire. If the cell is replaced by another cell then the balance point shifts to 65 cm then the emf of second cell is
(a) 3.05 V (b) 2.05 V (c) 4.05 V (d) 6.05 V

26. In a potentiometer the balancing with a cell is at length of 220 cm. On shunting the cell with a resistance of 3 Ω balance length becomes 130 cm. What is the internal resistance of this cell ?
(a) 4.5 Ω (b) 7.8 Ω (c) 6.3 Ω (d) 2.08 Ω

27. In a closed circuit, the current I (in ampere) at an instant of time t (in seconds) is given by $I = 4 - 0.08t$. The number of electrons flowing in 50 s through the cross-section of the conductor is
(a) 1.25×10^{19} (b) 6.25×10^{20}
(c) 5.25×10^{19} (d) 2.55×10^{20}

28. Two cells ε_1 and ε_2 connected in opposite to each other as shown in figure. The cell ε_1 is of emf 9 V and internal resistance 3 Ω the cell ε_2 is of emf 7 V and internal resistance 7 Ω. The potential difference between the points A and B is



(a) 8.4 V (b) 5.6 V (c) 7.8 V (d) 6.6 V

29. A wire connected in the left gap of a meter bridge balance a 10 Ω resistance in the right gap to a point which divides the bridge wire in the ratio 3 : 2. If the length of the wire is 1 m. The length of one ohm wire is
(a) 0.057 m (b) 0.067 m
(c) 0.37 m (d) 0.134 m

30. Three resistors of resistances 3 Ω, 4 Ω and 5 Ω are combined in parallel. This combination is connected to a battery of emf 12 V and negligible internal resistance, the current through each resistor in ampere are respectively
(a) 4, 3, 2.4 (b) 8, 7, 3.4
(c) 2, 5, 1.8 (d) 5, 5, 8.2

SOLUTION

1. **(b):** Radius of electron orbit, $r = 0.72$ Å
$= 0.72 \times 10^{-10}$ m
Frequency of revolution of electron in orbit of given atom
$\upsilon = \frac{1}{T}\ 9.4 \times 10^{18}$ rev/s
(where T is time period of revolution of electron in orbit)
∴ Then equivalent current is
$I = \frac{e}{T} = e\upsilon = 1.6 \times 10^{-19} \times 9.4 \times 10^{18} = 1.504$ A

2. **(d):** Given, $\varepsilon = 15$ V, $r = 4$ Ω, $I = 2$ A
Now, for resistance of the resistors
$\varepsilon - Ir = V = IR$
$15 - 2 \times 4 = 2 \times R$, $15 - 8 = 2R$
$R = \frac{7}{2} = 3.5$ Ω.
Terminal voltage of battery
$V = IR = 2 \times 3.5 = 7$ V

3. **(a)**

4. **(b):** Here, $R_0 = 5\ \Omega$, $R_{100} = 5.25\ \Omega$

$R_t = 5.5\ \Omega$

As $R_t = R_0(1 + \alpha t)$ $\therefore$ $R_{100} = R_0(1 + \alpha 100)$

$$\alpha = \frac{R_{100} - R_0}{R_0 \times 100} \quad \text{...(i)}$$

Let the temperature of hot bath be t°C

$R_t = R_0(1 + \alpha t)$

$$\alpha = \frac{R_t - R_0}{R_0 \times t} \quad \text{...(ii)}$$

Equating equations (i) and (ii), we get

$$\frac{R_{100} - R_0}{R_0 \times 100} = \frac{R_t - R_0}{R_0 \times t}$$

$$t = \frac{R_t - R_0}{R_{100} - R_0} \times 100 = \frac{5.5 - 5}{5.25 - 5} \times 100$$

$$= \frac{0.5}{0.25} \times 100 = 200°\text{C}$$

5. **(a):** When the heater is connected to the supply its initial current will be slightly higher than its steady value but due to heating effect of the current the temperature will rise. This cause an increase in resistance and a slight decrease in current to steady current.

6. **(a):** Potential difference across $20\ \Omega = 20 \times 1$ $= 20$ V = potential difference across R_2.

Current in $R_2 = 0.5$ A

$\therefore$ Resistance $R_2 = \dfrac{20\text{ V}}{0.5\text{ A}} = 40\ \Omega.$

Potential difference across $R_1 = 69$ V – 20 V = 49 V.

Current in $R_1 = 0.5\text{ A} + \dfrac{20}{10} + 1\text{ A} = 3.5\text{ A}.$

$\therefore$ $R_1 = \dfrac{49}{3.5} = 14\ \Omega$

7. **(b):** The resistance of rod before reformation

$$R_1 = R = \frac{\rho l_1}{\pi r_1^2} \quad \left[\because\ R = \frac{\rho l}{A} = \frac{\rho l}{\pi r^2}\right]$$

Now the rod is reformed such that

$$l_2 = \frac{l_1}{2}$$

$\therefore$ $\pi r_1^2 l_1 = \pi r_2^2 l_2$

(Given : volume remains constant)

or $$\frac{r_1^2}{r_2^2} = \frac{l_2}{l_1} \quad \text{... (i)}$$

Now $$R_2 = \frac{\rho l_2}{\pi r_2^2}$$

$$\therefore \quad \frac{R_1}{R_2} = \frac{\rho l_1}{\pi r_1^2} \Big/ \frac{\rho l_2}{\pi r_2^2} = \frac{l_1}{l_2} \times \frac{r_2^2}{r_1^2}$$

or $$\frac{R_1}{R_2} = \frac{l_1}{l_2} \times \frac{l_1}{l_2} = \left(\frac{l_1}{l_2}\right)^2 = (2)^2 \quad \text{(using (i))}$$

$$\frac{R_1}{R_2} = 4$$

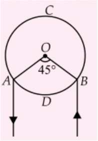
$$\therefore \quad R_2 = \frac{R_1}{4} = \frac{R}{4}$$

8. **(b):** The direction of current density is the direction of flow of positive charge in the circuit which is possible due to electric field produced by charges accumulated on the surface of wire.

9. **(a):** Resistance per unit length of ring,

$$\rho = \frac{R}{2\pi r}$$

Length of sections ADB and ACB are $r\theta$ and $r\ (2\pi - \theta)$



$\therefore$ Resistance of section ADB, $R_1 = \rho r\theta$

$$= \frac{R}{2\pi r} r\theta = \frac{R\theta}{2\pi}$$

and resistance of section ACB, $R_2 = \rho r(2\pi - \theta)$

$$= \frac{R}{2\pi r} r(2\pi - \theta) = \frac{R(2\pi - \theta)}{2\pi}$$

Now, R_1 and R_2 are connected in parallel between A and B then

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{\dfrac{R\theta}{2\pi} \times \dfrac{R(2\pi - \theta)}{2\pi}}{\dfrac{R\theta}{2\pi} + \dfrac{R(2\pi - \theta)}{2\pi}}$$

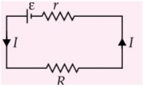
$$= \frac{\dfrac{R\theta \times R(2\pi - \theta)}{(2\pi)^2}}{\dfrac{R\theta + R(2\pi - \theta)}{2\pi}}$$

$$= \frac{R\theta(2\pi - \theta)}{4\pi^2}$$

Putting $\theta = 45° = \dfrac{\pi}{4}$ rad and $R = 15\ \Omega$

$$R_{eq} = \frac{15 \times \dfrac{\pi}{4} \times \left(2\pi - \dfrac{\pi}{4}\right)}{4\pi^2} = \frac{\dfrac{15\pi}{4}\left(\dfrac{7\pi}{4}\right)}{4\pi^2}$$

$$= \frac{105}{64}\ \Omega = 1.64\ \Omega$$

10. (b): 

Current in the circuit,

$$I=\frac{\varepsilon}{R+r}$$

Potential difference across R,

$$V=IR=\left(\frac{\varepsilon}{R+r}\right)R$$

$$V=\frac{\varepsilon}{1+\frac{r}{R}}$$

When $R = 0$, $V = 0$

$R = \infty$, $V = \varepsilon$

11. (b): Let emf be ε and internal resistance be r of the battery.

In first case,

$12 = \varepsilon - 2r$ ...(i)

In second case,

$15 = \varepsilon + 3r$ ...(ii)

Substract (ii) from (i), we get

$$r=\frac{3}{5}\ \Omega$$

Putting this value of r in eqn. (i), we get

$$\varepsilon=12+\frac{2\times3}{5}=\frac{60+6}{5}=\frac{66}{5}=13.2\text{ V}$$

12. (d): Current through 2 Ω, $I_1=\frac{2I}{3}$

Heat produced per second,

$$H_1=I_1^2\times2=\left(\frac{2I}{3}\right)^2\times2=\frac{8I^2}{9}$$

Current through 4 Ω, $I_2=\frac{I}{3}$

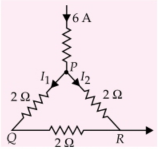
Heat produced per second

$$H_2=I_2^2\times4=\left(\frac{I}{3}\right)^2\times4=\frac{4I^2}{9}$$

Current through 3 Ω, = I

Heat produced $H_3 = I^2 \times 3 = 3I^2=\frac{27I^2}{9}$

$\therefore\ H_1 : H_2 : H_3 = 8 : 4 : 27.$

13. (a): 

Applying Kirchhoff's first law at the junction P, we get

$6 = I_1 + I_2$ ... (i)

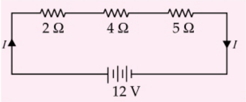
Applying Kirchhoff's second law to the closed loop $PQRP$, we get

$-2I_1 - 2I_1 + 2I_2 = 0$ or, $2I_1 + 2I_1 - 2I_2 = 0$

or, $4I_1 - 2I_2 = 0$ ... (ii)

Solve (i) and (ii), we get

$I_1 = 2$ A, $I_2 = 4$ A.

14. (d): 

Let current in the circuit is I. Then total resistance in the circuit

$R = R_1 + R_2 + R_3 = 2 + 4 + 5 = 11$

$\because\ V = IR$

$\therefore\ I=\frac{V}{R}=\frac{12}{11}$ A

The potential drop across 2 Ω resistance

$$V_1=IR_1=\frac{12}{11}\times2=2.18\text{ V}$$

The potential drop across 4 Ω resistance

$$V_2=IR_2=\frac{12}{11}\times4=4.36\text{ V}$$

The potential drop across 5 Ω resistance

$$V_3=IR_3=\frac{12}{11}\times5=5.45\text{ V}$$

Hence $(V_1, V_2, V_3) = (2.18, 4.36, 5.45)$ V

15. (c) : If one had considered a solid cylinder of radius b, one can suppose that it is made of two concentric cylinders of radius a and the outer part, joined along the length concentrically one inside the other.

If I_a and I_x are the currents flowing through the inner and outer cylinders I_{total}, $I_b = I_a + I_x$.

$$\frac{V}{R_b}=\frac{V}{R_a}+\frac{V}{R_x}$$

where R_b is the total resistance and R_x is the resistance of the tubular part.

$$\therefore\ \frac{1}{R_x}=\frac{1}{R_b}-\frac{1}{R_a}$$

But $R_a=\frac{\rho l}{\pi a^2}$ and $R_b=\frac{\rho l}{\pi b^2}$

$$\therefore\ \frac{1}{R_x}=\frac{\pi}{\rho l}(b^2-a^2)$$

$$\therefore\ R_x=\frac{\rho l}{\pi(b^2-a^2)}$$

16. (b) : Since $R = \rho \dfrac{l}{A}$,

$\therefore\ dR = \rho \dfrac{dl}{4\pi l^2}$.

(where l is any radius and dl is small element).
Total resistance,

$$R = \frac{\rho}{4\pi}\int_{r_1}^{r_2}\frac{dl}{l^2} = \frac{\rho}{4\pi}\left[-\frac{1}{l}\right]_{r_1}^{r_2} = \frac{\rho}{4\pi}\left[\frac{1}{r_1}-\frac{1}{r_2}\right].$$

$$R = \left[\frac{r_2 - r_1}{r_1 r_2}\right]\frac{\rho}{4\pi}.$$

17. (c) : Power, $P = \dfrac{V^2}{R}$

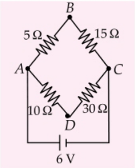
As the resistance of the bulb is constant

$\therefore\ \dfrac{\Delta P}{P} = \dfrac{2\Delta V}{V}$

$$\%\text{ decrease in power} = \frac{\Delta P}{P}\times 100 = \frac{2\Delta V}{V}\times 100$$

$$= 2 \times 2.5\% = 5\%$$

18. (a) : The given figure is a circuit of balanced Wheatstone bridge as shown in the figure.



Points B and D would be at the same potential, *i.e.*, $V_B - V_D = 0$ volt

19. (b): To get maximum equivalent resistance all resistances must be connected in series

$\therefore\ (R_{eq})_{\max} = R + R + R + \dots.\ nR. = nR$

To get minimum equivalent resistance all resistances must be connected in parallel.

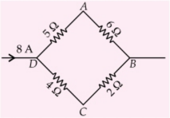
$$\therefore\ \frac{1}{(R_{eq})_{\min}} = \frac{1}{R} + \frac{1}{R} + \dots.\frac{1}{n}$$

$$\frac{1}{(R_{eq})_{\min}} = \frac{n}{R}$$

$$\Rightarrow\ (R_{eq})_{\min} = \frac{R}{n}\ \therefore\ \frac{(R_{eq})_{\max}}{(R_{eq})_{\min}} = \frac{nR}{R/n} = n^2$$

20. (a)

21. (b): Let I be the current flowing through arm DAB, then the current flowing through DCB will be $(8 - I)$



Then, $I(5 + 6)$
$= (8 - I)\ (4 + 2)$
$11I = 8 \times 6 - 6I$
$17I = 48$

$\therefore\ I = \dfrac{48}{17}$ A

Voltage across 5 Ω resistance

$$V_D - V_A = I \times R = \frac{48}{17}\times 5 = \frac{240}{17}\ \text{V} \quad \dots(i)$$

Voltage across 4 Ω resistance

$V_D - V_C = (8 - I) \times 4$

Subtracting (ii) from (i) we get, ...(ii)

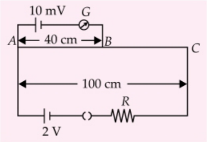
$$V_D - V_A - (V_D - V_C) = \frac{240}{17} - \frac{352}{17}$$

$\Rightarrow\ V_A - V_C = -6.6$ V

$\Rightarrow\ V_C - V_A = 6.6$ V

22. (a) : The current in the potentiometer wire AC is

$$I = \frac{2}{10 + R}$$



The potential difference across the potentiometer wire is

$$V = \text{current} \times \text{resistance} = \frac{2}{10+R}\times 10$$

The length of the wire is $l = 100$ cm.
So, the potential gradient along the wire is

$$k = \frac{V}{l} = \left(\frac{2}{10+R}\right)\times\frac{10}{100} \quad \dots(i)$$

The source of emf 10 mV is balanced against a length of 40 cm of the potentiometer wire

i.e. $10 \times 10^{-3} = k \times 40$

or $10\times 10^{-3} = \dfrac{2}{(10+R)}\times\dfrac{40}{10}$ (Using (i))

or $R = 790\ \Omega$.

23. (a) : When the same potential difference, that is the voltage, is applied as in houses,

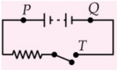
$\text{power} = VI = \dfrac{V^2}{R}$.

The smaller resistance consumes greater power. Here 100 W bulb has less resistance. It should

glow more brightly. The 60 W bulb has more resistance and therefore burns less brightly.

24. **(c) :** When the switch is not closed, a voltmeter connected across P and T will not show any potential difference. Between Q and T also there is no potential difference because circuit is not complete.

Therefore in both the cases, the voltmeter will read zero. Between P and Q, the emf of the battery will be given.



25. **(a) :** Here, in the balance condition of potentiometer

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{l_1}{l_2}$$

$\varepsilon_1 = 1.5$ V, $l_1 = 32$ cm, $l_2 = 65$ cm

$$\therefore \quad \varepsilon_2 = \varepsilon_1 \times \frac{l_2}{l_1}$$

$$= 1.5 \times \frac{65}{32} = 3.05 \text{ V}$$

26. **(d):** Here, $l_1 = 220$ cm, $l_2 = 130$ cm, $R = 3\ \Omega$

$$\therefore \quad \text{Internal resistance, } r = \left(\frac{l_1 - l_2}{l_2}\right)R$$

$$= \left(\frac{220-130}{130}\right) \times 3 = 2.08\ \Omega$$

27. **(b):** Let the number of electrons = N

$I = 4 - 0.08t$ A

$$\text{or} \quad \frac{dQ}{dt} = 4 - 0.08t \text{ A} \quad \text{or} \quad Q = \int_0^{50} (4 - 0.08t)dt \text{ C}$$

$$\text{or} \quad Ne = \left[4t - \frac{0.08t^2}{2}\right]_0^{50} = 100 \text{ C}$$

$$\text{or} \quad N = \frac{100}{e} = \frac{100}{1.6 \times 10^{-19}} = 6.25 \times 10^{20}$$

28. **(a) :** $I = \dfrac{\Delta\varepsilon}{r_1 + r_2} = \dfrac{9-7}{3+7} = \dfrac{2}{10} = 0.2$ A

Potential difference across $\varepsilon_1 = 9 - 0.2 \times 3$
$= 9 - 0.6$
$= 8.4$ V

Potential difference across ε_2,

$V_{AB} = \varepsilon_2 + 1\ r_2$
$= 7 + 0.2 \times 7 = 7 + 1.4$
$= 8.4$ V

29. **(b):** Here, $\dfrac{R}{S} = \dfrac{3}{2}$, $S = 10\ \Omega$

$$\therefore \quad R = \frac{3}{2} \times S = \frac{3}{2} \times 10 = 15\ \Omega$$

As the length of wire is 1 m.

$\therefore$ length of one ohm wire = 1/15 = 0.067 m

30. **(a) :** Since the voltage across the circuit is constant.

Then current through 3 Ω resistor

$$I_1 = \frac{V}{R_1} = \frac{12}{3} = 4 \text{ A}$$

The current through 4 Ω resistor

$$I_2 = \frac{V}{R_2} = \frac{12}{4} = 3 \text{ A}$$

and the current through 5 Ω resistor,

$$I_3 = \frac{V}{R_3} = \frac{12}{5} = 2.4 \text{ A}$$

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UNIT-7 Magnetic Effects of Current and Magnetism | Electromagnetic Induction and AC | Electromagnetic Waves

MAGNETIC FIELD

A magnet attracts small pieces of iron, cobalt, nickel etc. The space around a magnet within which its influence can be experience is called its magnetic field.

In 1820 Christian Oersted rediscovered; magnetic field is associated with electric current (*i.e.*, moving charges) and described as

- A moving charge or a current gets up creates a magnetic field in the space surrounding it.
- The magnetic field exerts a force on a moving charge or a current in the field.

Magnetic field is denoted by symbol B. It is also called magnetic induction or magnetic flux density.

Magnetic field is a vector quantity and its dimensional formula is $[ML^0T^{-2}A^{-1}]$.

The SI unit of magnetic field is tesla (T) or weber metre^{-2} (Wb m^{-2}). The CGS unit of magnetic field is gauss (G).

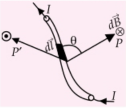
$1\text{ T} = 10^4\text{ G}$

Conventionally, the direction of the field perpendicular to the plane of the paper is represented by $\otimes$ if into the page, and by $\odot$ if out of the page.

BIOT SAVART'S LAW

To find the magnetic field at any point was first explained by Iean Biot (1774-1862) and Felix Savart (1791-1841) and proved it to be true in case of a straight conductor.

According to Biot Savart law, if $d\vec{B}$ is the magnetic field at point P, distant r from current element dl, then



- $dB \propto I$
- $dB \propto dl$
- $dB \propto \sin\theta$
- $dB \propto \dfrac{1}{r^2}$

Combining all these factors,

we get,

$$dB \propto \frac{Idl\sin\theta}{r^2} \text{ or } dB = K\frac{Idl\sin\theta}{r^2}$$

Here, K is proportionality constant and depends upon.

- the medium in which the conductor is placed.
- the system of units selected.

For space and SI units.

$$K = \frac{\mu_0}{4\pi} = 10^{-7}\text{ T m A}^{-1}(\text{or Wb m}^{-1}\text{A}^{-1})$$

Here μ_0 is a constant called permeability of free space.

$$\therefore \quad dB = \frac{\mu_0}{4\pi}\frac{Idlr\sin\theta}{r^3}$$

In vector form,

$$dB = \frac{\mu_0}{4\pi}I\frac{(d\vec{l}\times\vec{r})}{r^3}$$

Few points on Biot-Savart's law

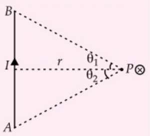
- This law is applicable only to infinitesimally small conductors.
- This law is analoyous to coulomb's law in electrostatics and as such is of fundamental importance.
- Applying the right hand rule for the vector product of two vectors. We find that $d\vec{B}$ is perpendicular to the plane containing $d\vec{l}$ and $\vec{r}$ is directed inwards . The situation is represented by a cross $\otimes$.
- For a point P' on the other side of $d\vec{l}$, the magnetic field $d\vec{B}$ is perpendicular to the plane of the upper and directed outwards. This situation is represented by a dot $\odot$.

- Just as the electric field produced by a charge distribution is the vector sum of the fields produced by the charges within the distribution, the magnetic field of any distribution of electric currents is the vector sum of the fields produced by small elements of that current distribution, *i.e.*, $\vec{B} = \sum d\vec{B}$.

Applications of Biot Savart's Law

Magnetic field due to a straight current carrying wire of finite length

Suppose a straight current carrying wire AB, carrying current I, lies in the plane of the paper.



The magnetic field at point P due to the wire is given by

$$B = \frac{\mu_0 I}{4\pi r}[\sin\theta_1 + \sin\theta_2]$$

Special cases:

- **For the wire of infinite length**

$$\theta_1 = \theta_2 = \frac{\pi}{2}$$

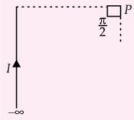
$$\therefore \quad B = \frac{\mu_0 I}{2\pi r}$$

- **Magnetic field at a point on the line of current**
  If a point lies on the line of current carrying wire then magnetic field at this point is always zero.

- **At point exactly in front of one end of semi-infinite wire**

Here $\theta_1 = 0$ and $\theta_2 = \dfrac{\pi}{2}$

$$\therefore \quad B = \frac{\mu_0 I}{4\pi r}$$

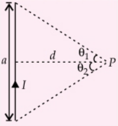


- **On perpendicular bisector of finite length**
  $a \rightarrow$ length of the wire
  $d \rightarrow$ perpendicular distance of the field point

Then,

$$\sin\theta_1 = \sin\theta_2 = \frac{a}{\sqrt{a^2 + 4d^2}}$$

$$\therefore \quad B = \frac{\mu_0 I}{4\pi d}\frac{2a}{\sqrt{a^2 + 4d^2}}$$



- **At a point not exactly in front of the end of a semi infinite wire**

Here $\theta_1 = \alpha$ and $\theta_2 = \dfrac{\pi}{2}$

$$\therefore \quad B = \frac{\mu_0 I}{4\pi r}(1 + \sin\alpha)$$

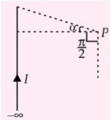
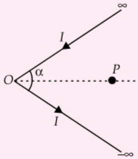
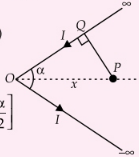


Illustration 1 : Find the magnetic field at point P shown in figure, the point P is on the bisector of angle between the wire.



Soln.: Assume $PO = x$, so
$PQ = x\sin(\alpha/2)$
Magnetic field B at P due to either segment of wire is,



$$B = \frac{\mu_0 I}{4\pi x\sin(\alpha/2)}\left[1 + \cos\frac{\alpha}{2}\right]$$

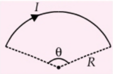
$\therefore$ Net magnetic field at P is

$$B_{net} = \frac{\mu_0 I}{2\pi x\sin(\alpha/2)}\left[1 + \cos\frac{\alpha}{2}\right]$$

Magnetic field due to a circular arc at the centre (Subtending an angle θ at the centre)

Consider a current element that subtends an angle θ as shown in figure. Magnetic field due to this element is

$$B = \frac{\mu_0 I\theta}{4\pi R} \quad \otimes$$



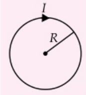
Special case :

- **Magnetic field at the centre of a loop**

Here the loop makes an angle

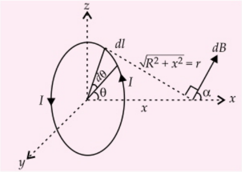
$\theta = 2\pi$ at the centre

$$\therefore \quad B = \frac{\mu_0 I}{2R}$$



- **Magnetic field at any point on the axis of a circular current carrying coil**
  Consider a circular conducting coil of radius R carrying current I. The loop lies on yz plane and its axis lies on x axis.
  From the symmetry of the system it can be seen

that diametrically opposite elements contribute to cancel the perpendicular components whereas parallel components are added up.



$\therefore \quad B = \int dB\cos\alpha$

Therefore the magnetic field at the distance x is

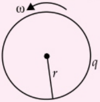
$$B = \frac{\mu_0 I R^2}{2(R^2+x^2)^{3/2}}$$

Note

- This magnetic field is directed along the axis of ring.
- The field strength is maximum at the centre (where $x = 0$)
  Magnetic field at the centre, $B = \frac{\mu_0 I}{2R}$
- At very large distance when $x >> R$, $B = \frac{\mu_0 I R^2}{2x^3}$

Illustration 2 : A conducting ring of radius r having charge q is rotating with angular velocity ω about its axis. Find the magnetic field at the centre of ring.

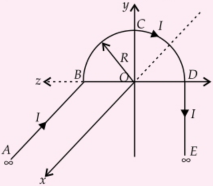
Soln.: Current in ring, $I = \frac{\omega q}{2\pi}$



Magnetic field $B = \frac{\mu_0 I}{2r} = \frac{\mu_0 \omega q}{2\pi \times 2r}$

$\Rightarrow \quad B = \frac{\mu_0 \omega q}{4\pi r}$

Illustration 3 : Find magnetic field at O, by the system of current carrying wire.



Soln.: As in figure, $\vec{B}_O = \vec{B}_{AB} + \vec{B}_{BCD} + \vec{B}_{DE}$

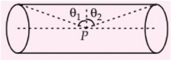
$$\Rightarrow \quad \vec{B}_O = \left(\frac{\mu_0 I}{4\pi R}\right)(-\hat{j}) + \left[\frac{\mu_0 I}{4R} + \frac{\mu_0 I}{4\pi R}\right](-\hat{i})$$

Magnetic Field Due to a Solenoid

A solenoid is a long cylindrical helix, which is obtained by winding closely a large number of turns of insulated wire over a tube of cardboard. When electric current is passed through it, a magnetic field is produced around and within the solenoid.

The magnetic field at a point P is given by

$$B = \frac{\mu_0 n I}{2}[\sin\theta_1 + \sin\theta_2]$$



which is directed along the axis.

where n is the number of turns per unit length.

Special cases :

- **For ideal solenoid (*i.e.* solenoid of infinite length) $R << L$**
  $\Rightarrow \quad B_m = \mu_0 n I \qquad$ (*i.e.* same everywhere)
- **Semi-infinitely long solenoid at the end**
  $$B = \frac{\mu_0 n I}{2}\left[\sin 0° + \sin\left(\frac{\pi}{2}\right)\right] = \frac{\mu_0 n I}{2}$$

AMPERE'S CIRCUITAL LAW

Ampere's circuital law is an alternative form of Biot-Savart law in magnetostatics. Ampere's circuital law gives a relation between the line integral of a magnetic field B and the total current I which produces this field.

This law states that "the line integral of the magnetic field $\vec{B}$ around any closed circuit is equal to μ_0 (permeability constant) times the total current I passing through this closed circuit".

Mathematically,

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I$$

In a simplified form, This law states that if field $\vec{B}$ is directed along the tangent to every point on the perimeter L of a closed curve and its magnitude is constant along the curve, then,

$$BL = \mu_0 I$$

Where I is the net current on closed by the closed circuit. The closed curve is called Amperean loop which is a geometrical entity and not a real wire loop.

Applications of Ampere's Law

Magnetic field due to infinite long cylindrical shell

In a infinitely long cylindrical shell carrying current I along the length and having inner and outer radii a and b respectively, the magnetic field at a point P at a distance

- $r > b$

$$B = \frac{\mu_0 I}{2\pi r}$$

- $r = b$

$$B = \frac{\mu_0 I}{2\pi b}$$

- $a < r < b$

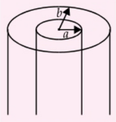
$$B = \frac{\mu_0 I(r^2 - a^2)}{2\pi r(b^2 - a^2)}$$

- $r = a$

$$B = 0$$

- $r < a$

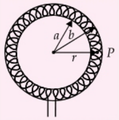
$$B = 0$$



Magnetic field due to toroid

Toroid is a structure formed by joining solenoid end to end.

The magnetic field at a point P at a distance r from the centre is given by



$$B = \frac{\mu_0 NI}{2\pi r}$$

$N \rightarrow$ Total number of turns.

Illustration 4 : Suppose that the current density in a wire of radius a varies with r according to Kr^2 where K is a constant and r is the distance from the axis of the wire. Find the magnetic field at a point at the distance r from the axis when (i) $r < a$ and (ii) $r > a$.

Soln.: Choose a circular path centered on the conductor's axis and apply Ampere's law.

(i) To find the current through the area enclosed by the path

$$dI = JdA = (Kr^2)(2\pi r dr)$$

$$i.e., \quad I = K\int_0^r 2\pi r^3 dr = \frac{\pi K r^4}{2}$$

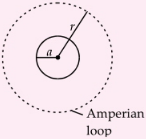
(i) Since $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$

$$\Rightarrow \quad B 2\pi r = \mu_0 \frac{\pi K r^4}{2} \Rightarrow B = \frac{\mu_0 K r^3}{4}$$

(ii) If $r > a$, then net current through the Amperian loop is

$$I' = \int_0^a Kr^2 2\pi r dr = \frac{\pi K a^4}{2}$$

Therefore $B = \dfrac{\mu_0 K a^4}{4r}$



MOTION OF CHARGED PARTICLES IN MAGNETIC FIELD

The electric charges moving in a magnetic field experience a force, while there is no such force on static charges. It was first recognized by Hendrik Antoon Lorentz.

When a charged particle of charge q, moving with velocity $\vec{v}$ in a uniform magnetic field $\vec{B}$, the force acting on it is $\vec{F} = q(\vec{v} \times \vec{B})$ or $F = qvB\sin\theta$.

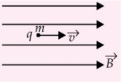
Here, θ is angle between $\vec{v}$ and $\vec{B}$.

The direction of $\vec{F}$ is perpendicular to both $\vec{v}$ and $\vec{B}$.

Cases of Projection :

- **Case I :** If velocity of charge particle $\vec{v}$ is parallel to $\vec{B}$, then

$$F = qvB\sin 0° \Rightarrow F = 0$$



The particle goes undeflected.

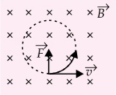
- **Case II :** If velocity of charge particle $\vec{v}$ is perpendicular to $\vec{B}$, then charge particle moves in a circular trajectory.

$$F = qvB\sin 90° \Rightarrow F = qvB$$

As path of charged particle in magnetic field is circular, we have

$$\frac{mv^2}{R} = qvB$$

Therefore,



- Radius of circle

$$R = \frac{mv}{qB}$$

- Angular velocity

$$\omega = \frac{v}{R} = \frac{qB}{m}$$

- Time period

$$T = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$$

- Frequency of revolution

$$\upsilon = \frac{qB}{2\pi m}$$

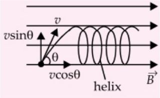
- **Case III :** If velocity $\vec{v}$ is at angle θ to $\vec{B}$, then the charge particle moves in helix.

Component of velocity parallel to magnetic field ($v\cos\theta$) remains constant and component perpendicular to magnetic field ($v\sin\theta$) is responsible for circular motion. Thus

- Radius of helix

$$\frac{m(v\sin\theta)^2}{R} = q(v\sin\theta)B$$

$$\Rightarrow \quad R = \frac{mv\sin\theta}{qB}$$



- Time period of revolution

$$T = \frac{2\pi R}{v\sin\theta}$$

$$\Rightarrow\ T = \frac{2\pi\left(\dfrac{mv\sin\theta}{qB}\right)}{v\sin\theta}\ \Rightarrow\ T = \frac{2\pi m}{qB}$$

- Pitch of helix

$$P_H = (v\cos\theta)(T)$$

$$\Rightarrow\ P_H = (v\cos\theta)\left(\frac{2\pi m}{qB}\right) = \frac{2\pi mv\cos\theta}{qB}$$

Lorentz Force

The total force experienced by a charged particle moving in a region where both electric and magnetic fields are present, is called Lorentz force.

A charge q in an electric field $\vec{E}$ experiences the electric force, $\vec{F}_e = q\vec{E}$.

$\vec{F}_e$ is independent of whether the charge q itself is moving or not.

Now, Magnetic force, $F_m = q(\vec{v}\times\vec{B})$

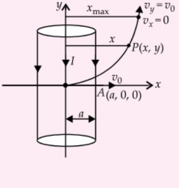
Thus, total force or the Lorentz force, experienced by the charge q due to both electric and magnetic fields is given by

$$\vec{F} = \vec{F}_e + \vec{F}_m = q(\vec{E} + \vec{v}\times\vec{B}).$$

Illustration 5 : From the surface of a round wire of radius a carrying a direct current I an electron escapes with a velocity v_0 perpendicular to the surface. Find what will be the maximum distance of the electron from the axis of the wire before it turns back due to the action of the magnetic field generated by the current.

Soln.: Here $B_A = \dfrac{\mu_0 I}{2\pi a}\hat{k}$

and $B_P = \dfrac{\mu_0 I}{2\pi x}\hat{k}$



Assume velocity of electron at P is

$$v_P = v_x\hat{i} + v_y\hat{j}$$

Here, $v_x^2 + v_y^2 = v_0^2$

$$\Rightarrow\ v_y^2 = v_0^2 - v_x^2$$

$$\Rightarrow\ v_y = \sqrt{v_0^2 - v_x^2}$$

The magnetic force on the electron at point P

$$\vec{F} = q(\vec{v}\times\vec{B}) = (-e)(v_x\hat{i} + v_y\hat{j})\times\left(\frac{\mu_0 I}{2\pi x}\hat{k}\right)$$

$$\Rightarrow\ \vec{F} = -\frac{ev_y\mu_0 I}{2\pi x}\hat{i} + \frac{ev_x\mu_0 I}{2\pi x}\hat{j}$$

$$\therefore\ F_x = -\frac{e\mu_0 I v_y}{2\pi x}\ \Rightarrow\ a_x = \frac{F_x}{m} = -\frac{e\mu_0 I v_y}{2\pi m x}$$

$$\text{or}\ \ v_x\cdot\frac{dv_x}{dx} = -\frac{e\mu_0 I}{2\pi m x}\sqrt{v_0^2 - v_x^2}$$

$$\frac{-v_x dv_x}{\sqrt{v_0^2 - v_x^2}} = \frac{e\mu_0 I}{2\pi m}\frac{dx}{x}$$

$$\Rightarrow\ \int_{v_0}^{0} -\frac{v_x dv_x}{\sqrt{v_0^2 - v_x^2}} = \frac{e\mu_0 I}{2\pi m}\int_a^{x_{max}}\frac{dx}{x}$$

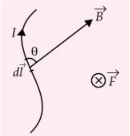
$$\Rightarrow\ x_{max} = ae^{\frac{2\pi m v_0}{\mu_0 I e}}$$

FORCE ON A CURRENT CARRYING CONDUCTOR IN A MAGNETIC FIELD

If a conductor carrying a current as placed in an external magnetic field. It experiences a mechanical force.

Force on current element $Id\vec{l}$, if placed in a magnetic field $\vec{B}$ is given by

$$d\vec{F} = I(d\vec{l}\times\vec{B})$$



Direction of magnetic force is determined by Fleming's left hand rule.

Total force on current carrying conductor of any arbitrary shape in uniform magnetic field $\vec{B}$ is

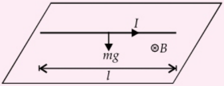
$$\vec{F} = \int I(d\vec{l}\times\vec{B}) = I\left(\int d\vec{l}\right)\times\vec{B} = I(\vec{l}\times\vec{B})$$

Illustration 6 : A straight wire of mass 200 g and length 1.5 m carries a current of 2 A. It is suspended in mid-air by a uniform horizontal magnetic field B. What is the magnitude of the magnetic field?

Soln.: We have that of mid-air suspension

$mg = IlB$

$$B = \frac{mg}{Il} = \frac{0.2\times 9.8}{2\times 1.5} = 0.65\ \text{T}$$



Force between Two Infinite Parallel Current Carrying Conductor

Two current carrying straight conductors if placed close to each other, they will exert (magnetic) force on each other.

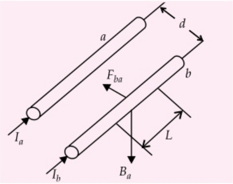


Figure shows two long parallel conductors a and b separated by a distance d and carrying currents I_a and I_b respectively. The conductor a produces a magnetic field B_a at all points along the conductor b. The right hand rule (figure) tells us that the direction of this field is downwards. Its magnitude is given by or from Ampere's circuital law,

$$B_a = \frac{\mu_0 I_a}{2\pi d}$$

The conductor b will experience a sideways force on account of the external field B_a. The direction of this force is towards the conductor a. The magnitude of this force is given by

$$F_{ba} = I_b L B_a = \frac{\mu_0 I_a I_b L}{2\pi d}$$

and force per unit length is

$$f_{ba} = \frac{\mu_0 I_a I_b}{2\pi d}$$

Parallel currents attract, and anti-parallel currents repel.

Illustration 7 : The horizontal component of the earth's magnetic field at a certain place is 3×10^{-5} T and the direction of the field is from the geographic south to the geographic north. A very long straight conductor is carrying a steady current of 1 A. What is the force per unit length on it when it is placed on a horizontal table and the direction of the current is

(a) east to west (b) south to north?

Soln.: $\vec{F} = I(\vec{L} \times \vec{B})$

$F = ILB \sin\theta$

The force per unit length is

$$f = \frac{F}{L} = IB\sin\theta$$

(a) When the current is flowing from east to west, $\theta = 90°$

Hence, $f = IB = 1 \times 3 \times 10^{-5} = 3 \times 10^{-5}$ N m^{-1}

The direction of the force is downwards. This direction may be obtained either by Fleming's left hand rule or the directional property of cross product of vectors.

(b) When the current is flowing from south to north, $\theta = 0° \Rightarrow f = 0$

Hence no force per unit length on the conductor.

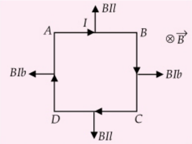
TORQUE ON A CURRENT CARRYING LOOP

We take uniform magnetic field in each case.

- **Case I : When plane of the loop is perpendicular to magnetic field**

  Length of $AB = DC = l$ and that of $BC = AD = b$

  Force experienced by all the sides are shown in the figure.



$\therefore$ Force on AB and DC are equal and opposite to the each other and that on BC and AD too.

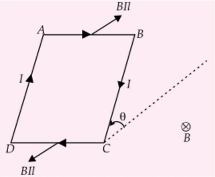
$\Rightarrow \sum\vec{F} = 0$

Since the line of action of the forces on AB and DC is same and also the line of action of the forces BC and AD is same, therefore torque is zero.

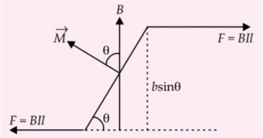
$\Rightarrow \vec{\tau} = 0$

- **Case II : When the plane of the loop is inclined to the magnetic field**

  In this case again $\sum\vec{F} = 0$



$\because$ Lines of action of the forces on AB and DC are different, therefore this forms a couple and produces a torque. Side view of the loop is shown in the figure.

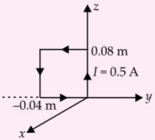


Torque = $BIl(b\sin\theta) = BI(lb)\sin\theta = BIA\sin\theta$
If loop has N turns, then
$\tau = BNIA\sin\theta$
In vector form
$\vec{\tau} = \vec{M} \times \vec{B}$, where $\vec{M} = NI\vec{A}$

where $\vec{M}$ is known as **magnetic moment** of the loop.

$\vec{A}$ is the area vector of the loop whose magnitude is area of the loop and direction is out of the plane for anti-clock wise sense of the current and into the loop for clockwise sense of current.

Illustration 8 : The rectangular coil having 100 turns is turned in a uniform magnetic field of $\frac{0.05}{\sqrt{2}}\hat{j}$ T as shown in the figure. Find the torque acting on the loop.



Soln.: The magnetic dipole moment of the current carrying coil is given by

$$\vec{M} = NIA\hat{n}$$

$$\vec{M} = 100 \times 0.5 \times 0.08 \times 0.04\,\hat{i}$$

$$\vec{M} = 16 \times 10^{-2}\,\hat{i}\text{ A m}^2$$

The torque acting on the coil is

$$\vec{\tau} = \vec{M} \times \vec{B} = MB\,(\hat{i} \times \hat{j})$$

$$\vec{\tau} = 16 \times 10^{-2} \times \frac{0.05}{\sqrt{2}}\hat{k} = 5.66 \times 10^{-3}\,\hat{k}\text{ N m}$$

MOVING COIL GALVANOMETER

It is an instrument used for the detection and measurement of small currents.

Principle of a moving coil galvanometer : When a current carrying coil is placed in a magnetic field, it experiences a torque.

In moving coil galvanometer the current I passing through the galvanometer is directly proportional to its deflection (θ).

$$I \propto \theta \quad \text{or,} \quad I = G\theta.$$

where $G = \frac{k}{NAB}$ = galvanometer constant

A = area of a coil, N = number of turns in the coil, B = strength of magnetic field, k = torsional constant of the spring *i.e.* restoring torque per unit twist.

Current Sensitivity

It is defined as the deflection produced in the galvanometer, when unit current flows through it.

$$I_s = \frac{\theta}{I} = \frac{NAB}{k}.$$

The unit of current sensitivity is rad A^{-1} or div A^{-1}.

Voltage Sensitivity

It is defined as the deflection produced in the galvanometer when a unit voltage is applied across the two terminals of the galvanometer.

$$V_s = \frac{\theta}{V} = \frac{\theta}{IR} = \frac{NAB}{kR}.$$

The unit of voltage sensitivity is rad V^{-1} or div V^{-1}.

$$V_s = \frac{1}{R}I_s.$$

AMMETER

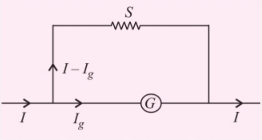
It is an instrument used to measure current in an electrical circuit.

Conversion of Galvanometer into a Ammeter

A galvanometer can be converted into an ammeter of given range by connecting a suitable low resistance S called shunt in parallel to the given galvanometer, whose value is given by

$$S = \left(\frac{I_g}{I - I_g}\right)G$$

where I_g is the current for full scale deflection of galvanometer, I is the current to be measured by the galvanometer and G is the resistance of galvanometer.



Ammeter is a low resistance instrument and it is always connected in series to the circuit. An ideal ammeter has zero resistance.

VOLTMETER

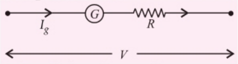
It is an instrument used to measure potential difference across any element in an electrical circuit.

Conversion of Galvanometer into Voltmeter

A galvanometer can be converted into voltmeter of given range by connecting a suitable resistance R in series with the galvanometer, whose value is given by

$$R = \frac{V}{I_g} - G$$

where V is the voltage to be measured, I_g is the current for full scale deflection of galvanometer and G is the resistance of galvanometer.



Voltmeter is a high resistance instrument and it is always connected in parallel with the circuit element across which potential difference is to be measured. An ideal voltmeter has infinite resistance.

In order to increase the range of an ammeter n times, the value of shunt resistance to be connected in parallel is $S = G/(n - 1)$.

In order to increase the range of voltmeter n times the value of resistance to be connected in series with galvanometer is $R = (n - 1)G$.

NATURAL MAGNET AND ARTIFICIAL MAGNET

Natural magnet : A natural magnet is an ore of iron (Fe_2O_4) which

- attracts small pieces of iron, cobalt and nickel towards it.
- when suspended freely, comes to rest along north-south direction.

Artificial magnet : Magnet which is prepared artifically is known as artificial magnets. *e.g.* a horse shoe magnet, a bar magnet, magnetic needle etc.

Properties of Magnets

Attractive property : When a magnet is dipped into iron filings it is found that the concentration of iron fillings, *i.e.* attracting power of the magnet is maximum at two points near the ends and minimum at the centre. The places in a magnet where its attracting power is maximum are known as poles while the place of minimum attracting power is known as the neutral region.

Directive property : When a magnet is suspended, its length becomes parallel to N-S direction. The pole at the end pointing north is known as north pole while the other pointing south is known as south pole.

Magnetic poles always exist in pairs *i.e.* an isolated magnetic pole does not exist.

Like poles repel each other and unlike poles attract each other.

Magnetic axis

The line joining the two poles of a magnet is known as magnetic axis.

Magnetic length

The distance between two poles along the axis of a magnet is known as magnetic length or effective length. As poles are not exactly at the ends, the effective length is lesser than the actual length of the magnet.

Magnetic Meridian

A vertical plane passing through the magnetic axis (*i.e.*, S - N line of a freely suspended magnet), is called the magnetic meridian.

MAGNETIC DIPOLE

A magnetic dipole consists of two unlike poles of equal strength and separated by a small distance *e.g.* a bar magnet, a compass needle etc. are magnetic dipoles.

Magnetic Dipole Moment

It is defined as the product of strength of either pole (m) and the magnetic length $(2\vec{l})$ of the magnet. It is denoted by $\vec{M}$.

Magnetic dipole moment = strength of either pole × magnetic length

$$\vec{M} = m(2\vec{l})$$

Magnetic dipole moment is a vector quantity and it is directed from south to north pole of the magnet.

The SI unit of magnetic dipole moment is A m^2.

Magnetic Field at a Point due to Magnetic Dipole (bar Magnet)

The magnetic field due to a bar magnet at any point on the axial line (end on position) is

$$B_{\text{axial}} = \frac{\mu_0}{4\pi}\frac{2Mr}{(r^2 - l^2)^2}$$

where r = distance between the centre of the magnet and the given point on the axial line, $2l$ = magnetic length of the magnet and M = magnetic moment of the magnet.

For short magnet $l^2 << r^2$

$$B_{\text{axial}} = \frac{\mu_0 2M}{4\pi r^3}$$

The direction of B_{axial} is along SN.

The magnetic field due to a bar magnet at any point on the equatorial line (board-side on position) of the bar magnet is

$$B_{\text{equatorial}} = \frac{\mu_0 M}{4\pi(r^2 + l^2)^{3/2}}$$

For short magnet

$$B_{\text{equatorial}} = \frac{\mu_0 M}{4\pi r^3}$$

The direction of $B_{\text{equatorial}}$ is parallel to NS.

Torque on a Magnetic Dipole Placed in a Uniform Magnetic Field

When a magnetic dipole of dipole moment $\vec{M}$ is placed in a uniform magnetic field $\vec{B}$, it will experience a torque and is given by

$\vec{\tau} = \vec{M} \times \vec{B}$ or $\tau = MB\sin\theta$

where θ is the angle between $\vec{M}$ and $\vec{B}$

Torque acting on a dipole is maximum ($\tau_{max} = MB$) when dipole is perpendicular to the field and minimum ($\tau = 0$) when dipole is parallel or antiparallel to the field.

When a dipole is placed in a uniform magnetic field, it will experience only torque and the net force on the dipole is zero while when it is placed in a non uniform magnetic field, it will experience both torque and net force.

Work done in Rotating the Magnetic Dipole in a Uniform Magnetic Field

Work done in rotating the magnetic dipole from θ_1 to θ_2 with respect to uniform magnetic field is

$$W = \int_{\theta_1}^{\theta_2} MB\sin\theta\, d\theta = -MB(\cos\theta_2 - \cos\theta_1) = MB(\cos\theta_1 - \cos\theta_2)$$

If the dipole is rotated from field direction *i.e.* $\theta_1 = 0°$ to position θ *i.e.* $\theta_2 = \theta$

$\therefore$ $W = MB(1 - \cos\theta)$.

Potential energy of a magnetic dipole

Potential energy of a magnetic dipole in a uniform magnetic field is

$U = -\vec{M}\cdot\vec{B} = -MB\cos\theta$

The potential energy of the dipole will be minimum ($= -MB$) when $\theta = 0°$, *i.e.*, the dipole is parallel to the field, and maximum ($= MB$) when $\theta = 180°$, *i.e.*, the dipole is antiparallel to the field.

Current loop as a magnetic dipole

A current loop behaves as a magnetic dipole whose magnetic dipole moment is $M = IA$ where A is the area enclosed by loop and I is the current flowing in the loop.

If there are N turns in a loop, then $M = NIA$.

Gauss's law for magnetism

Gauss's law for magnetism states that the net magnetic flux through any closed surface is zero.

$$\phi = \sum_{\substack{\text{all area}\\ \text{elements } \Delta S}} \vec{B}\cdot\Delta\vec{S} = 0$$

This law establishes that isolated magnetic poles do not exist.

ELEMENTS OF THE EARTH'S MAGNETIC FIELD

Three quantities are needed to specify the magnetic field of the earth on its surface – the horizontal component, the magnetic declination and the magnetic dip. These are known as elements of the earth's magnetic field or magnetic elements.

Magnetic declination : Magnetic declination at a place is defined as the angle between the geographic meridian and magnetic meridian.

Magnetic dip or inclination : Magnetic dip at a place is defined as the angle made by the earth's magnetic field with the horizontal in the magnetic meridian. It is denoted by δ.

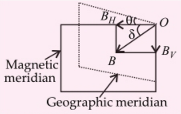
Horizontal component : It is component of earth's magnetic field along the horizontal direction in the magnetic meridian. It is denoted by B_H.

If B is intensity of earth's total magnetic field, then the horizontal component of earth's magnetic field is given by

$$B_H = B\cos\delta$$

Also, the vertical component of earth's magnetic field,

$B_V = B\sin\delta$



$\therefore$ $B = \sqrt{B_H^2 + B_V^2}$

and $\tan\delta = \dfrac{B_V}{B_H}$

The earth always has a vertical component except at equator.

The earth always has a horizontal component except at the poles.

In a vertical plane at an angle θ to magnetic meridian

$B'_H = B_H\cos\theta$ and $B'_V = B_V$

$\therefore$ $\tan\delta' = \dfrac{B'_V}{B'_H} = \dfrac{B_V}{B_H\cos\theta} = \dfrac{\tan\delta}{\cos\theta}$

$\tan\delta' = \dfrac{\tan\delta}{\cos\theta}$

If at a given place δ_1 and δ_2 are angles of dip in two arbitrary vertical planes which are perpendicular to each other, the true angle of dip δ is given by

$\cot^2\delta = \cot^2\delta_1 + \cot^2\delta_2$

Angle of dip δ at a plane is related to its magnetic latitude λ through the relation

$\tan\delta = 2\tan\lambda$

MAGNETIC INTENSITY

When a magnetic material is placed in a magnetic field, it becomes magnetised. The capability of the magnetic field to magnetise a material is expressed by means of a magnetic vector $\vec{H}$, called the magnetic intensity of the field. The relation between magnetic induction B and magnetising field H is

$$B = \mu H$$

where μ is the permeability of medium.

It is a vector quantity and its SI unit is A m^{-1}.

Intensity of magnetisation

It is defined as the magnetic moment per unit volume.

$$I = \frac{\text{Magnetic moment}}{\text{Volume}} = \frac{M}{V}$$

If A = uniform area of cross-section of the magnetised specimen (a rectangular bar)
$2l$ = magnetic length of the specimen
m = strength of each pole of the specimen,

$$\therefore \quad I = \frac{m \times 2l}{A \times 2l} = \frac{m}{A}$$

The intensity of magnetisation is a vector quantity and its SI unit is A m^{-1}. Its dimensional formula is $[M^0L^{-1}T^0A]$.

Magnetic susceptibility

It is defined as the ratio of the intensity of magnetisation (I) to the magnetising field.

$$\chi_m = \frac{I}{H}$$

It is a scalar quantity with no units and dimensions. Physically, it represents the ease with which a magnetic material can be magnetised. *i.e.*, large value of χ_m implies that the material is more susceptible to the field and hence can be easily magnetised.

Magnetic permeability

It is defined as the ratio of magnetic induction to the magnetising field (H).

$$\mu = \frac{B}{H}$$

It is a scalar quantity having unit H m^{-1} and its dimensional formula is $[MLT^{-2}A^{-2}]$.

It measures the degree to which a magnetic material can be penetrated or permeated by the magnetising field.

Relative permeability : It is defined as ratio of permeability of a medium to that of free space

$$\mu_r = \frac{\mu}{\mu_0}$$

It has no units and dimensions.

Relationship between magnetic permeability and suspectibility

$$\mu_r = 1 + \chi_m \quad \text{with} \quad \mu_r = \frac{\mu}{\mu_0}$$

CLASSIFICATION OF MAGNETIC MATERIALS

On the basis of magnetic properties, different materials have been classified into three categories :

- Diamagnetic substances
- Paramagnetic substances
- Ferromagnetic substances

Diamagnetic substances : Diamagnetic substances are those in which the individual atoms/molecules/ions do not posses any net magnetic moment on their own. When such substances are placed in an external magnetic field, they get feebly magnetised in a direction opposite to the magnetic field. *e.g.* antimony, bismuth, copper, lead, gold, silver, water, alcohol, mercury, air, hydrogen, nitrogen and all inert gases like helium, neon, argon etc.

The relative permeability of diamagnetic substances is less than one. The magnetic susceptibility of diamagnetic substances is small and negative. The magnetic susceptibility of diamagnetic substances is independent of temperature.

Paramagnetic substances : Paramagnetic substances are those in which each individual atom/molecule/ion has a net non zero magnetic moment of its own. When such substances are placed in an external magnetic field, they get feebly magnetised in the direction of the magnetic field.

The relative permeability of paramagnetic substances is just greater than one. The magnetic susceptibility of paramagnetic substances is small and positive. The magnetic susceptibility of paramagnetic substance is dependent on the temperature and it varies with temperature according to the given equation

$$\chi_m = \frac{C}{T}$$

This is known as Curie law. The constant C is known as Curie's constant.

Ferromagnetic substances : Ferromagnetic substances are those in which each individual atom/molecule/ion has a non zero magnetic moment, as in a paramagnetic substance. When such substances are placed in an external magnetic field, they get strongly magnetised in the direction of the field. *e.g.* iron, cobalt, nickel, gadolinium and a number of their alloys.

The relative magnetic permeability of ferromagnetic substances is very large. The magnetic susceptibility of ferromagnetic substance is positive and very large. The ferromagnetic property depends on temperature. At high temperature, ferromagnetic becomes paramagnet. The temperature of transition from ferromagnetic to paramagnetic is known as Curie temperature (T_C). The susceptibility above the Curie temperature *i.e.* in the paramagnetic phase is given by

$$\chi_m = \frac{C}{T - T_C} \; (T > T_C)$$

This is known as Curie-Weiss law.

HYSTERESIS

Hysteresis is the phenomenon of lagging of magnetic induction (B) or intensity of magnetisation (I) behind the magnetising field (H), when a specimen is taken through a cycle of magnetisation. From the hysteresis loop of material, we can study about retentivity, coercivity etc. of the material. The study of these characteristics enables us to select suitable materials for different purposes.

Type of ferromagnetic materials

Ferromagnetic materials are divided into two types :

- **Soft magnetic materials :** These have low retentivity, low coercivity and small hysteresis loss. These are used for making electromagnets, cores or transformers, motors and generators. Soft iron, mu-metal and stalloy are examples of these materials
- **Hard magnetic materials :** These have high retentivity, high coercivity and large hysteresis-loss. These are used in making permanent magnets of various kinds of electric meters and loudspeakers. Steel, alnico, alcomax and ticonal are examples of ferromagnetic materials of these materials.

MAGNETIC FLUX

The magnetic flux through any surface placed in a magnetic field is the total number of magnetic lines of force crossing this surface normally.

It is measured as the product of the component of the magnetic field normal to the surface and the surface area.

Magnetic flux is a scalar quantity, denoted by ϕ or ϕ_B.

- If a uniform magnetic field $\vec{B}$ passes normally through a plane surface area A, then

  $\phi = BA$
- If the field $\vec{B}$ makes angle θ with the normal drawn to the area A, then

  $\phi = B\cos\theta \times A$

  $$\phi = BA\cos\theta = \vec{B}\cdot\vec{A}$$

The SI unit of magnetic flux is **weber.** The CGS unit is **maxwell.**

1 weber = 10^8 maxwell

Also 1 weber = 1 tesla × metre^2.

The dimensions of magnetic flux are

$$[\phi] = [M^1L^2T^{-2}A^{-1}]$$

Characteristics of Magnetic Flux

The magnetic flux can be positive , negative or zero depending on the angle θ. For $\theta = 90°$, $\cos\theta = 0$ and $\phi = 0$. Thus, whenever the angle between area vector and magnetic field is 90°, the flux is zero. *i.e.* whenever the plane of the surface is parallel to $\vec{B}$, the flux is zero. The flux is positive for $0° \le \theta < 90°$ and negative for $90° < \theta \le 180°$. The magnetic flux is taken as negative if field lines enter the area and positive if field lines leave the area.

ELECTROMAGNETIC INDUCTION

The phenomenon of production of induced emf and hence induced current, due to a change of magnetic flux linked with a closed circuit is called electromagnetic induction

Laws of Electromagnetic Induction

- Faraday's laws gives the magnitude of induced emf.
- Lenz's law gives the direction of induced emf.

Faraday's Laws of Electromagnetic Induction

When magnetic flux passing through a loop changes with time or magnetic lines of force are cut by a conducting wire then an emf is produced in the loop or in that wire. This emf is called induced emf.

If the circuit is closed then the current will be called induced current.

The magnitude of induced emf is equal to the rate of change of flux w.r.t. time in case of loop. In case of a wire it is equal to the rate at which magnetic lines of force are cut by a wire.

$$\varepsilon = -\frac{d\phi}{dt} \quad \text{...(i)}$$

(–) sign indicates that the emf will be induced in such a way that it will oppose the change of flux.

Some points:

The induced emf in a circuit does not depend on the resistance of the circuit as $\varepsilon = -(d\phi/dt)$. However, the induced current in the circuit does depend on the resistance.

$$I = \frac{\varepsilon}{R} = -\frac{1}{R}\left(\frac{d\phi}{dt}\right)$$

The induced charge that flows in the circuit depends on the change of flux only and not on how fast or slow the flux changes.

$$\frac{dq}{dt} = -\frac{1}{R}\left(\frac{d\phi}{dt}\right) \text{ or } dq = -\frac{d\phi}{R}$$

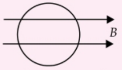
On integrating , the total charge that flows in the circuit is found to be

$$q = \frac{(\phi_1 - \phi_2)}{R}$$

If the number of turns in the coil is N, then the charge that flows through the coil is

$$q = \frac{N(\phi_1 - \phi_2)}{R}$$

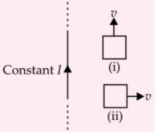
Illustration 9 : A coil is placed in a constant magnetic field. The magnetic field is parallel to the plane of the coil as shown in figure. Find the emf induced in the coil.



Soln.: $\phi = 0$ (always) since area is perpendicular to magnetic field.

$\therefore$ emf = 0

Illustration 10 : Figure shows a long current carrying wire and two rectangular loops moving with velocity v. Find the direction of current in each loop.



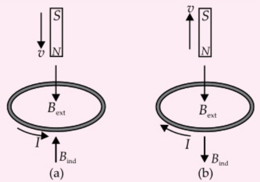
Soln.: In loop (i) no emf will be induced because there is no flux change.
In loop (ii) emf will be induced because the coil is moving in a region of decreasing magnetic field inward direction. Therefore to oppose the flux decrease in inward direction, current will be induced such that its magnetic field will be inwards. For this direction of current should be clockwise.

Lenz's Law

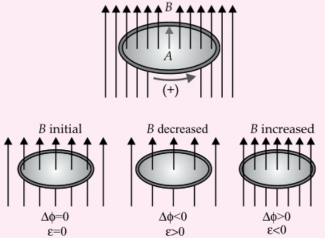
In 1833, German physicist Hemrich Lenz gave general law. It states that the direction of induced current in a circuit is such that it opposes the causes or the change which produces it.

In figure (a), as the magnet approaches, the (positive) flux through the coil increases. The induced current sets up an induced magnetic field, B_{ind}, whose (negative) flux opposes this change. The direction of B_{ind} is opposite to that of external field, B_{ext}, due to the magnet.

In figure (b), the induced current sets up an induced field whose (positive) flux opposes the decrease in flux of B_{ext}. In this case, the induced magnetic field points in the same direction as the external field.



In order to incorporate Lenz's law into Faraday's law, we need a sign convention for the induced emf. First we choose the direction of the vector area to make the initial flux positive. The right-hand rule, in this case with the thumb along B and the fingers curled around the loop, tells us whether clockwise or counterclockwise is the positive sense, as shown in figure. Figure shows that the sign of the emf is always opposite to the sign of the change in flux $\Delta\phi$.



This feature can be incorporated into Faraday's law by including a negative sign. The proportionality in equation (i) can now be turned into an equation with an appropriate constant of proportionality that depends on the system of units. In SI units, the constant is unity. Therefore, the modern statement of Faraday's law of electromagnetic induction is

$$\varepsilon = -\frac{d\phi}{dt}$$

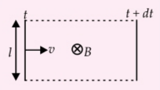
Suppose that the loop is replaced by a coil with N turns. If the flux through each turn is the same, each turn has the same induced emf. Since all these emf's are in the same sense, the net emf induced in a coil with N turns is

$$\varepsilon = -N\frac{d\phi}{dt}$$

where ϕ is the flux through each turn.

MOTIONAL EMF

We can find emf induced in a moving rod by considering the number of lines cut by it per second assuming there are B lines per unit area . Thus when a rod of length l moves with velocity v in a magnetic field B, as shown, it will sweep area per unit time equal to lv and hence it will cut Blv lines per unit time.



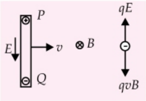
Hence emf induced between the ends of the rod = Blv

Also emf = $\frac{d\phi}{dt}$. Here ϕ denotes flux passing through the area, swept by the rod. The rod sweeps an area equal to $lvdt$ in time interval dt. Flux through this area = $Blvdt$. Thus $\frac{d\phi}{dt} = \frac{Blvdt}{dt} = Blv.$

Explanation of emf induced in rod on the basis of magnetic force

If a rod is moving with velocity v in a magnetic field

B, as shown , the free electrons in a rod will experience a magnetic force in downward direction and hence free electrons will accumulate at the lower end and there will be a deficiency of free electrons and hence a surplus of positive charge at the upper end. These charges at the ends will produce an electric field in downward direction which will exert an upward force on electron.



If the rod has been moving for quite some time enough charges will accumulate at the ends so that the two forces qE and qvB will balance each other. Thus $E = vB$.

$V_P - V_Q = vBl$

The moving rod is equivalent to the following diagram, electrically.

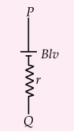
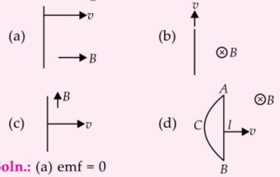


Illustration 11 : Find the emf induced in the rod in the following cases.



Soln.: (a) emf = 0

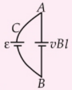
(b) emf = 0

(c) emf = 0

(d) Figure shows a closed coil $ABCA$ moving in a uniform magnetic field B with a velocity v. The flux passing through the coil is a constant and therefore the induced emf is zero.

Now consider rod AB, which is a part of the coil.

Emf induced in the rod = Blv



Suppose the emf induced in part ACB is ε, as shown.

Since the emf in the coil is zero,

emf (in ACB) + emf (in BA) = 0

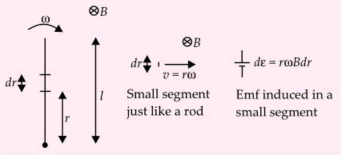
or $\quad -\varepsilon + vBl = 0 \quad$ or $\quad \varepsilon = vBl$

Thus emf induced in any path joining A and B is same, provided the magnetic field is uniform. Also the equivalent emf between A and B is Blv (here the two emfs are in parallel).

Induced emf Due to Rotation

Rotation of the rod

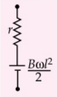
Consider a conducting rod of length l rotating in a uniform magnetic field.



Emf induced in a small segment of length dr, of the rod = $vBdr = r\omega Bdr$

$\therefore$ Emf induced in the rod $= \omega B\int_0^l r\,dr = \frac{1}{2}B\omega l^2$

Alternatively

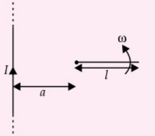


or $\quad \varepsilon = \frac{d\phi}{dt}$

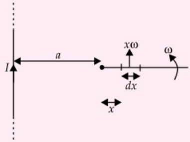
$$= \frac{\text{Flux through the area swept by the rod in time } dt}{dt}$$

$$= \frac{B\frac{1}{2}l^2\omega dt}{dt} = \frac{1}{2}B\omega l^2$$

Illustration 12 : A rod of length l is rotating with an angular speed ω about its one end which is at a distance a from an infinitely long wire carrying current I. Find the emf induced in the rod at the instant shown in the figure.



Soln.: Consider a small segment of rod of length dx, at a distance x from one end of the rod.



Alternatively

$$d\varepsilon = \frac{\mu_0 I}{2\pi(x+a)}(x\omega)dx$$

$$\therefore \quad \varepsilon = \int_0^l \frac{\mu_0 I}{2\pi(x+a)}(x\omega)dx = \frac{\mu_0 I\omega}{2\pi}\left[l - a\ln\left(\frac{l+a}{a}\right)\right]$$

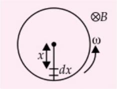
Emf Induced in a Rotating Disc

Consider a disc of radius r rotating in a magnetic field B.

Consider an element dx at a distance x from the centre. This element is moving with speed $v = \omega x$.

$\therefore$ Induced emf across dx

$= B(dx)v = B(dx)\omega x = B\omega x dx$



$\therefore$ Emf between the centre and the edge of disc,

$$= \int_0^r B\omega x dx = \frac{B\omega r^2}{2}$$

Fixed Loop in a Varying Magnetic Field

Now consider a circular loop, at rest in a varying magnetic field. Suppose the magnetic field is directed inside the page and it is increasing in magnitude. The emf induced in the loop will be

$$\varepsilon = -\frac{d\phi}{dt}$$

Flux through the coil will be

$$\phi = -\pi r^2 B;$$

$$\frac{d\phi}{dt} = -\pi r^2 \frac{dB}{dt} \qquad \therefore\ \varepsilon = \pi r^2 \frac{dB}{dt}$$

$$\therefore\ E 2\pi r = \pi r^2 \frac{dB}{dt} \quad \text{or} \quad E = \frac{r}{2}\frac{dB}{dt}$$

Thus changing magnetic field produces electric field which is non conservative in nature. The lines of force associated with this electric field are closed curves.

Eddy currents

The circulating currents set up in bulk pieces of metal moving through a magnetic field are called eddy currents.

These are also called foucault currents.

The direction of eddy currents is given by Lenz's law, or Fleming's right hand rule.

Some of the important applications of eddy currents are : electromagnetic damping, induction furnace, electromagnetic brakes, induction motor, speedometers and in diathermy, *i.e.*, deep heat treatment of parts of human body.

INDUCTOR

An inductor is a device for storing energy in a magnetic field. An inductor is generally called as inductance. In usual practice a coil or solenoid is treated as inductor. It is denoted by symbol "-ꝏꝏ-".

SELF INDUCTION

Whenever the current passing through a coil or circuit changes, the magnetic flux linked with it will also change. As a result of this, an emf is induced in the coil or the circuit. This phenomenon is called self induction and the emf induced is called self induced emf or back emf.

If a current I flows through a coil and ϕ is the magnetic flux linked with the coil, then

$\phi \propto I$ or $\phi = LI$

where L is coefficient of self induction or self inductance of the coil.

The self induced emf is

$$\varepsilon = -\frac{d\phi}{dt} = -L\frac{dI}{dt}$$

The SI unit of L is henry (H) and its dimensional formula is $[ML^2T^{-2}A^{-2}]$.

Self inductance of a long solenoid is $L = \mu_0 n^2 l A$ where l is length of the solenoid, n is number of turns per unit length of a solenoid and A is area of cross section of the solenoid.

Self inductance of a circular coil is

$$L = \frac{\mu_0 N^2 \pi R}{2}$$

where R is the radius of the coil and N is the number of turns.

MUTUAL INDUCTION

Whenever the current passing through a coil or circuit changes, the magnetic flux linked with a neighbouring coil or circuit will also change. Hence an emf will be induced in the neighbouring coil or circuit. This phenomenon is called mutual induction. The coil or circuit in which the current changes is known as primary while the other in which emf is set up is known as secondary.

Let I_P be the current flowing through primary coil at any instant. If ϕ_S is the flux linked with secondary coil then

$\phi_S \propto I_P$ or $\phi_S = MI_P$

where M is coefficient of mutual inductance of two coils.

The emf induced in the secondary coil is given by

$$\varepsilon_S = -M\frac{dI_P}{dt}$$

The SI unit of M is henry (H) and its dimensional formula is $[ML^2T^{-2}A^{-2}]$.

Coefficient of coupling (K) : Coefficient of coupling of two coils is a measure of the coupling between the two coils and is given by

$$K = \frac{M}{\sqrt{L_1 L_2}}$$

where L_1 and L_2 are coefficients of self inductance of the two coils and M is coefficient of mutual inductance of the two coils.

The coefficient of mutual inductance M of two long coaxial solenoid is given by $M = \mu_0 n_1 n_2 \pi r_1^2 l$.

where l is the length of each solenoid, n_1 is number of turns per unit length of inner solenoid, n_2 is the number of turns per unit length of outer solenoid, r_1 is the radius of inner solenoid, r_2 is the radius of outer solenoid.

Combination of Inductances

Two inductors of inductances L_1 and L_2 are connected in series and are kept apart so that their mutual inductance is negligible, then the equivalent inductance of the combination is given by

$L_S = L_1 + L_2$.

Two inductors of self-inductance L_1 and L_2 are connected in series and they have mutual inductance M, then the equivalent inductance of the combination is given by

$L = L_1 + L_2 \pm 2M$.

The plus sign occurs if windings in the two coils are in the same sense, while minus sign occurs if windings are in opposite sense.

Two inductors of inductances L_1 and L_2 are connected in parallel and are kept far apart so that mutual inductance between them is negligible, then their equivalent inductance is given by

$$L = \frac{L_1 L_2}{L_1 + L_2} \quad \text{or} \quad \frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$$

Illustration 13 : Two coils have mutual inductance of 1.5 H. If the current in primary circuit is raised to a value of 50 A in one second after closing the circuit, what is the induced emf in the secondary?

Soln.: Mutual inductance, $M = 1.5$ H.

Rate of change of current in the primary, *i.e.*, $\frac{dI_P}{dt}$ = 50 A/s.

Let ε be the induced emf in the secondary,

then $\varepsilon = M\frac{dI_P}{dt} = -1.5 \times 50$

= – 70 V.

ENERGY STORED IN AN INDUCTOR

When a current I flows through an inductor, the energy stored in it, is given by

$$U = \frac{1}{2}LI^2$$

The energy stored in an inductor is in the form of magnetic energy.

GROWTH AND DECAY OF CURRENT IN LR CIRCUIT

During growth, the instantaneous current in the circuit is given by

$I = I_0\,(1 - e^{-Rt/L}) = I_0(1 - e^{-t/\tau})$

where I_0 is the maximum value of current, $\tau = L/R$ = time constant of LR circuit.

During decay, the instantaneous current in the circuit is given by

$I = I_0 e^{-Rt/L} = I_0 e^{-t/\tau}$

CHARGING AND DISCHARGING OF A CAPACITOR THROUGH RESISTOR

During charging the instantaneous charge on the capacitor is given by

$q = q_0(1 - e^{-t/RC}) = q_0(1 - e^{-t/\tau})$

where q_0 is the maximum value of charge, $\tau = RC$ is the time constant of CR circuit.

During discharging, the instantaneous charge on the capacitor in given by

$q = q_0 e^{-t/RC} = q_0 e^{-t/\tau}$

Illustration 14 : A capacitor of capacitance 0.5 μF is discharged through a resistance of 10 MΩ. Find the time taken for half the charge on the capacitor to escape.

Soln.: Here, $C = 0.5\ \mu F = 0.5 \times 10^{-6}\ F = 5 \times 10^{-7}$ F

$R = 10\ M\Omega = 10^7\ \Omega$

Time constant, $\tau = CR = (5 \times 10^{-7}\ F)(10^7\ \Omega) = 5$ s

If t is time taken for half the charge on the capacitor to escape, then charge left after time t is equal to $Q_0/2$, *i.e.*,

$$Q = \frac{Q_0}{2} \text{ or } \frac{Q}{Q_0} = \frac{1}{2}$$

As $\quad Q = Q_0 e^{-t/\tau}, \quad \frac{Q}{Q_0} = e^{-t/\tau}$

or $\quad 2 = e^{t/\tau}$ or $\frac{t}{\tau} = \ln 2$

or $\quad t = (\ln 2)\tau = (0.6931)(5s) = 3.5$ s

ALTERNATING CURRENT

An alternating current is that current whose magnitude changes continuously with time and direction reverses periodically where as that current which flows with a constant magnitude in the same direction is known as direct current.

Most of the electric power generated and used in the world is in the form of AC.

Alternating current is represented by

$I = I_0 \sin\omega t$ or $I = I_0 \cos\omega t$

Here, I is the instantaneous value of current *i.e.*, magnitude of current at any instant of time t and I_0 is the peak value or maximum value of AC. It is also called amplitude of AC, ω is called angular frequency of AC.

$$\omega = \frac{2\pi}{T} = 2\pi\upsilon$$

The terms used for AC hold equally for alternating emf which is represented by

$V = V_0 \sin\omega t$

or $V = V_0 \cos\omega t$.

Average Values of AC Voltage and AC Current

AC voltage or current are commonly sinusoidal (sine or cosine function) and their mean values for complete cycle is zero. The average values for half cycles are equally positive and negative.

Average value for one half cycle (or rectified average value)

$$V = V_0 \sin\omega t$$

$$\therefore (V)_{\text{av}} = \frac{\int_0^{T/2} V\,dt}{\int_0^{T/2} dt} = \frac{2}{T}\int_0^{T/2} V_0 \sin\omega t\,dt = \frac{2}{\pi}V_0 = 0.637\,V_0.$$

This is also known as the rectified average value of a sinusoidal voltage and is represented as V_{av}.

Root Mean Square Value (V_{rms} or I_{rms})

Since V and I are equally negative and positive, their squares will always be positive and the square root of the average of their squares is will give the rms values.

$\therefore \quad V = V_0 \sin\omega t$

$$(V^2)_{\text{av}} = \frac{1}{T}\int_0^T V_0^2 \sin^2\omega t\,dt = \frac{V_0^2}{2T}\int_0^T (1-\cos 2\omega t)dt = \frac{V_0^2}{2}.$$

Thus $V_{\text{rms}} = \sqrt{(V^2)_{\text{av}}} = \dfrac{V_0}{\sqrt{2}}$

and $I_{\text{rms}} = \sqrt{(I^2)_{\text{av}}} = \dfrac{I_0}{\sqrt{2}}$

or $\text{RMS value} = \dfrac{\text{Peak value}}{\sqrt{2}}$

Illustration 15 : If a domestic appliance draws 2.5 A from a 220 V, 60 Hz power supply, find

(a) the average current
(b) the average of the square of the current
(c) the current amplitude
(d) the supply voltage amplitude.

Soln.: (a) The average of sinusoidal AC values over any whole number of cycles is zero.

(b) RMS value of current $I_{\text{rms}} = 2.5$ A

$\therefore \quad (I^2)_{\text{av}} = (I_{\text{rms}})^2 = 6.25\ \text{A}^2$

(c) $I_{\text{rms}} = \dfrac{I_0}{\sqrt{2}}$

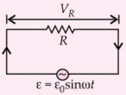
$\therefore$ current amplitude $= \sqrt{2}\,I_{\text{rms}} = \sqrt{2}(2.5\ \text{A}) = 3.5\ \text{A}$

(d) $V_{\text{rms}} = 220\ \text{V} = \dfrac{V_0}{\sqrt{2}}$

$\therefore$ Supply voltage amplitude
$V_0 = \sqrt{2}(V_{\text{rms}}) = \sqrt{2}(220\ \text{V}) = 311\ \text{V}.$

Series AC Circuit

When only resistance is in AC circuit



According to Kirchhoff's loop law, at any instant, the algebraic sum of the potential difference around a closed loop in a circuit must be zero.

$\varepsilon - V_R = 0$ or $\varepsilon - I_R R = 0$

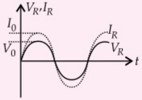
$\varepsilon_0 \sin\omega t - I_R R = 0$

$$I_R = \frac{\varepsilon_0}{R}\sin\omega t = I_0 \sin\omega t \qquad \text{...(i)}$$

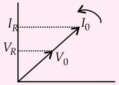
where I_0 is the maximum current, $I_0 = \dfrac{\varepsilon_0}{R}$.

From above equation, we see that the instantaneous voltage drop across the resistor is

$$V_R = I_0 R \sin\omega t \qquad \text{... (ii)}$$



Wave diagram
(a)

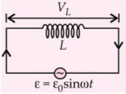


Phasor diagram
(b)

We see in equation (i) and (ii), I_R and V_R both vary as $\sin\omega t$ and reach their maximum values at the same time as shown in figure (a), they are said to be in phase. A phasor diagram is used to represent phase relationships. The lengths of the arrows correspond to V_0 and I_0. The projections of the arrows onto the vertical axis gives V_R and I_R.

In case of the single-loop resistive circuit, the current and voltage phasors lie along the same line, as shown in figure (b), because I_R and V_R are in phase.

When only inductor is in AC circuit



The induced emf across the inductor is given by $L\dfrac{dI}{dt}$.

On applying Kirchhoff's loop rule to the circuit,

$$\varepsilon - V_L = 0 \quad \Rightarrow \quad \varepsilon - L\frac{dI}{dt} = 0.$$

When we rearrange this equation and substitute $\varepsilon = \varepsilon_0 \sin\omega t$, we get

$$L\frac{dI}{dt} = \varepsilon_0 \sin\omega t \qquad \text{...(iii)}$$

Integration of this expression gives the current as a function of time

$$I_L = \frac{\varepsilon_0}{L}\int \sin\omega t\,dt = -\frac{\varepsilon_0}{\omega L}\cos\omega t + C$$

For average value of current over one time period to be zero, $C = 0$

$$\therefore \quad I_L = -\frac{\varepsilon_0}{\omega L}\cos\omega t$$

When we use the trigonometric identity $\cos\omega t = -\sin(\omega t - \pi/2)$, we can express equation as

$$I_L = \frac{\varepsilon_0}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right) \qquad \text{...(iv)}$$

From equation (iv), we see that the current reaches its maximum value when $\cos\omega t = 1$

$$I_0 = \frac{\varepsilon_0}{\omega L} = \frac{\varepsilon_0}{X_L} \qquad \text{...(v)}$$

where the quantity X_L, called the inductive reactance, is $X_L = \omega L$.

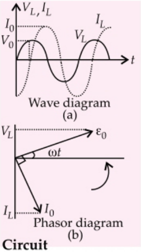
The expression for the rms current is similar to equation (v), with ε_0 replaced by ε_{rms}. Inductive reactance, like resistance, has unit of ohm.

$$V_L = L\frac{dI}{dt} = \varepsilon_0 \sin\omega t$$
$$= I_0 X_L \sin\omega t$$

We can think of equation (v) as ohms law for an inductive circuit. On comparing result of equation (iv) with equation (iii), we can see that the current and voltage are out of phase with each other by $\pi/2$ rad, or 90°.

A plot of voltage and current versus time is given in figure (a).

The voltage reaches its maximum value one quarter of an oscillation period before the current reaches its maximum value. The corresponding phasor diagram for this circuit is shown in figure (b). Thus, we see that for a sinusoidal applied voltage, the current in an inductor always lags behind the voltage across the inductor by 90°.



Wave diagram
(a)
Phasor diagram
(b)

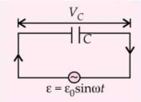
When only conductor is in AC Circuit

Apply Kirchoff's loop rule to the circuit

$\varepsilon - V_C = 0$

$V_C = \varepsilon = \varepsilon_0 \sin\omega t \qquad$ ...(vi)

where V_C is the instantaneous voltage drop across the capacitor.



From the definition of capacitance, $V_C = Q/C$, and this value for V_C substituted into equation (vi) gives $Q = C\varepsilon_0 \sin\omega t$

Since $I = dQ/dt$, on differentiating above equation gives the instantaneous current in the circuit, *i.e.*,

$$I_C = \frac{dQ}{dt} = C\varepsilon_0 \omega \cos\omega t.$$

Here again we see that the current is not in phase with the voltage drop across the capacitor, given by equation (vi). Using the trigonometric identity $\cos\omega t = \sin(\omega t + \pi/2)$, we can express this equation in the alternative form

$$I_C = \omega C\varepsilon_0 \sin\left(\omega t + \frac{\pi}{2}\right) \qquad \text{...(vii)}$$

From equation (vii), we see that the current in the circuit reaches its maximum value when $\cos\omega t = 1$.

$$I_0 = \omega C\varepsilon_0 = \frac{\varepsilon_0}{X_C}$$

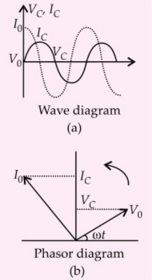
where $X_C = \dfrac{1}{\omega C}$ is called the capacitive reactance

The SI unit of X_C is also ohm. The rms current is given by an expression similar to equation with V_0 replaced by V_{rms}.

Combining equation (vi) and (vii), we can express the instantaneous voltage drop across the capacitor as $V_C = V_0 \sin\omega t = I_0 X_C \sin\omega t$.

Comparing the result of equation (vii) with equation (vi), we see that the current is $\dfrac{\pi}{2}$ rad or 90° out of phase with the voltage across the capacitor.

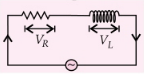
A plot of current and voltage versus time, shows that the current reaches its maximum value one quarter of a cycle sooner than the voltage reaches its maximum value. The corresponding phasor diagram is shown in the figure (b). Thus we see that for a sinusoidally applied emf, the current always leads the voltage across a capacitor by 90°.



Wave diagram
(a)
Phasor diagram
(b)

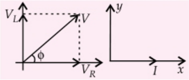
Series L-R Circuit

Now consider an AC circuit consisting of a resistor of resistance R and an inductor of inductance L in series with an AC source generator.



Suppose in phasor diagram, current is taken along positive x-direction. Then V_R is also along positive x-direction and V_L along positive y-direction.

As we know potential difference across a resistance in AC is in phase



with current, and it leads current in phase by 90° across the inductor, so we can write

$$V = V_R + jV_L = IR + j(IX_L) = IR + j(I\omega L) = IZ$$

Here, $Z = R + jX_L = R + j(\omega L)$ is called impedance of the circuit. Impedance plays the same role in AC circuits as the ohmic resistance does in dc circuits. The modulus of impedance is,

$$|Z| = \sqrt{R^2 + (\omega L)^2}.$$

The potential difference leads the current by an angle,

$$\phi = \tan^{-1}\left|\frac{V_L}{V_R}\right| = \tan^{-1}\left(\frac{X_L}{R}\right)$$

or $$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

Illustration 16 : An alternating voltage of 220 V r.m.s. at a frequency of 40 cycles/second is supplied to a circuit containing a pure inductance of 0.01 H and a pure resistance of 6 Ω in series. Calculate (a) the current, (b) potential difference across the resistance, (c) potential difference across the inductance, (d) the time lag.

Soln.: The impedance of *L-R* series circuit is given by

$$Z = [R^2 + (\omega L)^2]^{1/2} = [(R)^2 + (2\pi\upsilon L)^2]^{1/2}$$
$$= [6^2 + (2 \times 3.14 \times 40 \times 0.01)^2]^{1/2} = 6.504\ \Omega$$

(a) R.M.S. value of current

$$I_{rms} = \frac{\varepsilon_{rms}}{Z} = \frac{220}{6.504} = 33.83\ \text{A}$$

(b) The potential difference across the resistance is given by

$$V_R = I_{rms} \times R = 33.83 \times 6 = 202.98\ \text{V}$$

(c) Potential difference across the inductance is given by

$$V_L = I_{rms} \times (\omega L) = 33.83 \times (2 \times 3.14 \times 40 \times 0.01) = 84.98\ \text{V}$$

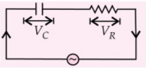
(d) Phase angle $\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$

$$\therefore \quad \phi = \tan^{-1}(0.4189) = 22°46'$$

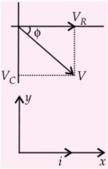
$$\text{Time lag} = \frac{\phi}{360} \times T = \frac{\phi}{360} \times \frac{1}{\upsilon} = \frac{22°46'}{360 \times 40}$$
$$= 0.001579\ \text{s}$$

Series R-C Circuit

Consider an AC circuit consisting of a resistor of resistance R and a capacitor of capacitance C in series with an AC source generator.



Suppose in phasor diagram current is taken along positive x-direction. Then V_R is along positive x-direction but V_C is along negative y-direction as potential difference across a capacitor in AC lags in phase by 90° with the current in the circuit. So, we can write



$$V = V_R - jV_C = IR - j(IX_C)$$
$$= IR - j\left(\frac{I}{\omega C}\right) = IZ$$

Here, impedance is, $Z = R - j\left(\frac{1}{\omega C}\right)$

The modulus of impedance is,

$$|Z| = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

and the potential difference lags the current by an angle,

$$\phi = \tan^{-1}\left|\frac{V_C}{V_R}\right| = \tan^{-1}\left(\frac{X_C}{R}\right) = \tan^{-1}\left(\frac{1/\omega C}{R}\right) = \tan^{-1}\left(\frac{1}{\omega RC}\right)$$

Illustration 17 : An A.C. source of angular frequency ω is fed across a resistor R and a capacitor C in series. The current registered is I. If now the frequency of the source is changed to ω/3 (but maintaining the same voltage), the current in the circuit is found to be halved. Calculate the ratio of reactance to resistance at the original frequency.

Soln.: At angular frequency ω, the current in *R-C* circuit is given by

$$I_{rms} = \frac{\varepsilon_{rms}}{\sqrt{R^2 + \frac{1}{(\omega C)^2}}} \quad \text{...(i)}$$

When frequency is changed to ω/3, the current is halved. Thus

$$\frac{I_{rms}}{2} = \frac{\varepsilon_{rms}}{\sqrt{R^2 + \frac{1}{(\omega/3)^2 C^2}}} = \frac{\varepsilon_{rms}}{\sqrt{R^2 + (9/\omega^2 C^2)}} \quad \text{...(ii)}$$

From equation (i) and (ii),

we have $$\frac{1}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} = \frac{2}{\sqrt{R^2 + \frac{9}{\omega^2 C^2}}}$$

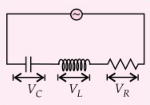
solving this equation, we get $3R^2 = \frac{5}{\omega^2 C^2}$

Hence, the ratio of reactance to resistance is

$$\frac{(1/\omega C)}{R} = \sqrt{\frac{3}{5}}$$

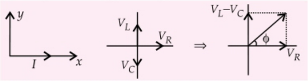
SERIES L-C-R CIRCUIT AND RESONANCE

Now consider an AC circuit consisting of a resistor of resistance R, a capacitor of capacitance C and an inductor of inductance L, in series



with an AC source generator.

Suppose in a phasor diagram, current is taken along positive x-direction. Then V_R is along positive x-direction. V_L along positive y- direction and V_C along negative y-direction, as potential difference across an inductor leads the current by 90° in phase while that across a capacitor, lags it in phase by 90°.



$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

So, we can write, $V = V_R + jV_L - jV_C$

$$= IR + j(IX_L) - j(IX_C)$$

$$= IR + j[I(X_L - X_C)] = IZ$$

Here impendance is,

$$Z = R + j(X_L - X_C) = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

The modulus of impedance is,

$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$ and the potential difference leads the current by an angle

$$\phi = \tan^{-1}\left|\frac{V_L - V_C}{V_R}\right| = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

$$= \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right) \quad \text{...(viii)}$$

The steady current in the circuit is given by

$$I = \frac{V_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \sin(\omega t + \phi)$$

where ϕ is given from equation (viii)

The peak current is $I_0 = \dfrac{V_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$

It depends on angular frequency ω of AC source and it will be maximum when $\omega L - \dfrac{1}{\omega C} = 0 \Rightarrow \omega = \sqrt{\dfrac{1}{LC}}$

and corresponding frequency is $\upsilon = \dfrac{\omega}{2\pi} = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC}}$.

This frequency is known as **resonant frequency** of the given circuit. At this frequency, peak current will be $I_0 = \dfrac{V_0}{R}$.

If the resistance R in the LCR circuit is zero, the peak current at resonance is $I_0 = \dfrac{V_0}{0} = \infty$.

It means, there can be a finite current in pure LC circuit even without any applied emf, This current in the circuit is at frequency,

$$\upsilon = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

Illustration 18 : A resistance R, and inductance L and a capacitor C all are connected in series with an A.C. supply. The resistance R is 16 Ω and for a given frequency, the inductive reactance L is 24 Ω and capacitive reactance C is 12 Ω. If the current in the circuit is 5 A, find

(a) the potential difference across R, L and C
(b) the impedance of the circuit
(c) the voltage of A.C supply
(d) phase angle

Soln.: (a) Potential difference across resistance

$V_R = IR = 5 \times 16 = 80$ V

Potential difference across inductance

$V_L = I \times (\omega L) = 5 \times 24 = 120$ V

Potential difference across condenser

$V_C = I \times (1/\omega C) = 5 \times 12 = 60$ V

(b) $$Z = \sqrt{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]}$$

$$= \sqrt{(16)^2 + (24-12)^2} = 20\ \Omega$$

(c) The voltage of A.C. supply is given by $R = IZ = 5 \times 20 = 100$ V

(d) $$\phi = \tan^{-1}\left[\frac{\omega L - (1/\omega C)}{R}\right]$$

$$= \tan^{-1}\left[\frac{24-12}{16}\right] = \tan^{-1}(0.75) = 36°46'$$

ELECTROMAGNETIC WAVES

In 1865, Maxwell theoretically predicted the existence of electromagnetic waves. An electromagnetic wave is a wave radiated by an accelerated charge which propagates through space as coupled electric and magnetic fields, oscillating perpendicular to each other and to the direction of propagation of the wave.

In 1888, Heinrich Hertz demonstrated the production and detection of electromagnetic waves by electrical means.

Characteristics of Electromagnetic Waves

Electric and magnetic fields oscillate sinusoidally in space and time in an electromagnetic wave. The oscillating electric and magnetic fields are perpendicular to each other, and to the direction of propagation of the electromagnetic wave. The electric and magnetic fields have the same frequency of oscillation and are in the same phase.

For an electromagnetic wave of frequency υ, wavelength λ, propagating along + z-direction, the electric and magnetic fields can be written as

$E = E_x(t) = E_0 \sin(kz - \omega t)$

BRAIN MAP

OSCILLATIONS

Simple Harmonic Motion

Characteristics of SHM

- General equation of displacement of particle executing linear SHM
  $y = A\sin(\omega t + \phi)$
- Time period of SHM, $T = \frac{1}{\upsilon} = \frac{2\pi}{\omega}$
- Velocity, $v = \omega\sqrt{A^2 - y^2}$
- Acceleration, $a = -\omega^2 y$

Maximum Velocity and Acceleration in SHM

- At mean position velocity is maximum
  $v_{max} = A\omega$
- Acceleration is maximum at extreme position
  $a_{max} = -\omega^2 A$

Dynamics of SHM

- $\vec{F} = -m\omega^2\vec{x}$ or $\vec{F} = -k\vec{x}$
  where, $k = m\omega^2$ = force constant
- Angular velocity, $\omega = \sqrt{\frac{k}{m}}$
- Time period, $T = 2\pi\sqrt{\frac{m}{k}}$
- Frequency, $\upsilon = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

Energy in Linear SHM

- Kinetic energy
  $$K = \frac{1}{2}m\omega^2(A^2 - y^2) = \frac{1}{2}m\omega^2 A^2\cos^2\omega t$$
- Potential energy
  $$U = \frac{1}{2}m\omega^2 y^2 = \frac{1}{2}m\omega^2 A^2\sin^2\omega t$$
- Total energy
  $$E = \frac{1}{2}m\omega^2 A^2 = \frac{2\pi^2 mA^2}{T^2}$$
- At mean position kinetic energy is maximum, at extreme position potential energy is maximum and total energy is constant at every position during simple harmonic motion.

Differential Equations of SHM

- For linear SHM, $\frac{d^2y}{dt^2} + \omega^2 y = 0$
- For angular SHM, $\frac{d^2\theta}{dt^2} + \omega^2\theta = 0$

Angular SHM

- Angular displacement,
  $\theta = \theta_0\sin(\omega t + \delta)$
- Torque, $\tau = -k\theta$
- Angular velocity, $\omega = \sqrt{k/I}$
- Angular acceleration, $\alpha = -\frac{k\theta}{I}$
- Time period of oscillation, $T = 2\pi\sqrt{\frac{I}{k}}$
- Frequency of oscillation, $\upsilon = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{I}}$

Energy in Angular SHM

- Potential energy, $U = \frac{1}{2}k\theta^2 = \frac{1}{2}I\omega^2\theta^2$
- Kinetic energy, $K = \frac{1}{2}I\omega^2$
- Total energy, $E = \frac{1}{2}I\omega^2\theta_0^2$

SHM in Spring

- Equation of motion
  $$\frac{d^2y}{dt^2} = \frac{-ky}{m} = -\omega^2 y$$
- If the spring is not light but has a definite mass then
  $$T = 2\pi\sqrt{\frac{m + \frac{m_s}{3}}{k}}$$
- Two bodies of masses m_1 and m_2 are attached through a light spring of spring constant k, the time period of oscillation
  $$T = 2\pi\sqrt{\frac{\mu}{k}}$$
  where $\mu = \frac{m_1 m_2}{m_1 + m_2}$
- If the mass m attached to a spring oscillates in a non viscous liquid of density σ then
  $$T = 2\pi\left[\frac{m}{k}\left(1 - \frac{\sigma}{\rho}\right)\right]^{1/2}$$
  where ρ is the density of suspended mass.
- The force of gravity has no effect on force constant k and time period of oscillating mass.

Oscillations of Loaded Spring Combinations

- For two springs of spring factors k_1 and k_2 connected in parallel, effective spring factor
  $k = k_1 + k_2$ and $T = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$
- For two springs connected in series,
  $$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} \quad \text{or} \quad k = \frac{k_1 k_2}{k_1 + k_2}$$
  and $T = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}}$
- When length of a spring made n times, its spring factor becomes $1/n$ times and hence time period increases $\sqrt{n}$ times
- When a spring is cut into n equal pieces, spring factor of each part becomes nk and
  $$T = 2\pi\sqrt{\frac{m}{nk}}$$

Simple Pendulum

- Time period $T = 2\pi\sqrt{\frac{l}{g}}$
- If the length of simple pendulum is very large,
  $$T = 2\pi\sqrt{\frac{1}{g\left(\frac{1}{l} + \frac{1}{R}\right)}}$$
  where R is the radius of length of pendulum
- If a simple pendulum oscillates in a non viscous liquid of density σ then its time period
  $$T = 2\pi\left[\frac{l}{\left(1 - \frac{\sigma}{\rho}\right)g}\right]^{1/2}$$
  where ρ is the density of suspended mass.
- When a pendulum is kept in a car which is sliding down then
  $$T = 2\pi\sqrt{\frac{l}{g\cos\theta}}$$
  where θ is the angle of inclination.

Physical Pendulum

- Time period of physical pendulum
  $$T = 2\pi\sqrt{\frac{I}{mgd}}$$
  where d is the distance from centre of gravity of rigid body to pivoked point.

Damped and Forced Oscillations

- Angular frequency of the damped oscillation
  $$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$
  where b is damping constant.
- Mechanical energy of the damped oscillator
  $$E(t) = \frac{1}{2}kA^2 e^{-bt/m}$$
- Amplitude of forced oscillations when driving frequency is far from natural frequency
  $$A = \frac{F_0}{m(\omega^2 - \omega_d^2)}$$
- When driving frequency is close to natural frequency, *i.e.*, at resonance,
  $$A = \frac{F_0}{\omega_d b}$$

Time Period of Different SHM's

- A plank of mass m and area of cross section A is floating in a liquid of density ρ when depressed, it starts oscillating then
  $$T = 2\pi\sqrt{\frac{m}{\rho Ag}}$$
- In case of water oscillating in U-tube, then $T = 2\pi\sqrt{\frac{h}{g}}$
  where h is the height of liquid column in each limb
- A ball of mass m is made to oscillate in the neck of an air chamber having volume V and neck area a then
  $$T = 2\pi\sqrt{\frac{mV}{Ba^2}}$$
  where B = bulk modulus of elasticity in air.
- A small ball of radius r is rolling down in a hemispherical bowl of radius R, then
  $$T = 2\pi\sqrt{\frac{R - r}{g}}$$
  where R is the radius of bowl and r is the radius of ball
- For a body executing SHM in a tunnel dug along any chord of earth.
  $$T = 2\pi\sqrt{\frac{R_e}{g}} = 84.6\text{ min}$$
- Time period of torsional pendulum $T = 2\pi\sqrt{\frac{I}{C}}$ where $C = \frac{\pi\eta r^4}{2l}$
  where, η = modulus of elasticity of wire
  r = radius of wire
  l = length of wire

$$= E_0 \sin\left[2\pi\left(\frac{z}{\lambda} - \upsilon t\right)\right] = E_0 \sin\left[2\pi\left(\frac{z}{\lambda} - \frac{t}{T}\right)\right]$$

$B = B_y(t) = B_0 \sin(kz - \omega t)$

$$= B_0 \sin\left[2\pi\left(\frac{z}{\lambda} - \upsilon t\right)\right] = B_0 \sin\left[2\pi\left(\frac{z}{\lambda} - \frac{t}{T}\right)\right]$$

The **amplitudes** of electric and magnetic fields in electromagnetic waves are related as

$$E_0 = cB_0 \quad \text{or} \quad B_0 = \frac{E_0}{c}$$

The speed of electromagnetic waves in free space is given by

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

where μ_0 and ε_0 are the permeability and permittivity of free space.

The speed of electromagnetic waves in a material medium is given by

$$v = \frac{1}{\sqrt{\mu\varepsilon}}$$

where μ and ε are permeability and permittivity of the medium.

Also, $v = \dfrac{1}{\sqrt{\mu_0 \mu_r \varepsilon_0 \varepsilon_r}} = \dfrac{c}{\sqrt{\mu_r \varepsilon_r}}$.

These waves do not require any medium for their propagation.

These waves do not carry any charge.

These waves are not deflected by electric and magnetic fields.

They travel with the speed of light c (= 3×10^8 m s^{-1}) in vacuum.

The frequency of electromagnetic waves does not change when they travel from one medium to another but their wavelength changes.

These waves are transverse in nature, hence can be polarised.

Illustration 19 : The electric field of an electromagnetic wave travelling through vacuum is given by the equation $E = E_0 \sin(kx - \omega t)$. The quantity that is independent of wavelength is

(a) $\dfrac{k}{\omega}$ (b) ωk

(c) k (d) ω

Soln.: (a) $\dfrac{k}{\omega} = \dfrac{\frac{2\pi}{\lambda}}{2\pi\upsilon} = \dfrac{1}{c}$

Energy Density of Electromagnetic Waves

Electromagnetic waves carry energy as they travel through space and this energy is equally shared by electric and magnetic fields.

The energy density of the electric field is

$$u_E = \frac{1}{2}\varepsilon_0 E^2$$

The energy density of magnetic field is

$$u_B = \frac{1}{2}\frac{B^2}{\mu_0}$$

Average energy density of the electric field is

$$< u_E > = \frac{1}{4}\varepsilon_0 E_0^2$$

Average energy density of the magnetic field is

$$< u_B > = \frac{1}{4}\frac{B_0^2}{\mu_0} = \frac{1}{4}\varepsilon_0 E_0^2$$

Total average energy density of electromagnetic wave is

$$< u > = < u_E > + < u_B >$$

$$= 2 < u_E > = 2 < u_B > = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{1}{2}\frac{B_0^2}{\mu_0}$$

The units of $<u_E>$ and $<u_B>$ are J m^{-3}.

Intensity of Electromagnetic Wave

It is defined as rate of flow of energy through unit area perpendicular to the direction of propagation of wave.

The intensity of electromagnetic wave is given by

$$I = < u > c = \frac{1}{2}\varepsilon_0 E_0^2 c = \frac{1}{2}\frac{B_0^2 c}{\mu_0}$$

Illustration 20 : The electric field of a plane electromagnetic wave of amplitude 2 V m^{-1} propagating along z-axis varies with time. The average energy density of the magnetic field is (in J m^{-3})

Soln.: Amplitude of electric field and magnetic field are related by the relation, $\dfrac{E_0}{B_0} = c$

Average energy density of the magnetic field is

$$< u_B > = \frac{1}{4}\frac{B_0^2}{\mu_0} = \frac{1}{4}\frac{E_0^2}{\mu_0 c^2} \qquad \left(\because \; B_0 = \frac{E_0}{c}\right)$$

$$= \frac{1}{4}\varepsilon_0 E_0^2 \qquad \left(\because \; c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}\right)$$

$$= \frac{1}{4} \times 8.854 \times 10^{-12} \times (2)^2$$

$$= 8.854 \times 10^{-12} \text{ J m}^{-3}$$

$$\approx 8.86 \times 10^{-12} \text{ J m}^{-3}$$

Momentum of Electromagnetic Wave

An electromagnetic wave carries linear momentum.

Electromagnetic wave strikes the surface at normal incidence and transports a total energy U to the surface in a time t, if the surface absorbs all the incident energy, the total momentum p transported to the surface is

$p = \dfrac{U}{c}$ (complete absorption)

If the surface is a perfect reflector and incidence is normal then the momentum transported to the surface is

$p = \dfrac{2U}{c}$ (complete reflection)

Radiation Pressure

It is defined as the pressure exerted by the electromagnetic wave on a surface.

If I is the intensity of the incident electromagnetic radiation, then the radiation pressure for normal incidence is

- $P_{\text{radiation}} = \frac{I}{c}$ (perfectly absorbing surface)
- $P_{\text{radiation}} = \frac{2I}{c}$ (perfectly reflecting surface)

POYNTING VECTOR

In electromagnetic wave, the rate of flow of energy through a unit area perpendicular to the direction of propagation wave is described by a vector called poynting vector.

The poynting vector is given by

$$\vec{S} = \frac{1}{\mu_0}\left(\vec{E}\times\vec{B}\right)$$

In magnitude, $S = \frac{1}{\mu_0}|\vec{E}\times\vec{B}| = \frac{EB}{\mu_0}$

The direction of poynting vector is along the direction of the propagation of wave.

The SI unit of poynting vector is W m^{-2}.

Average value of poynting vector is

$$\langle S \rangle = \frac{1}{2}\varepsilon_0 E_0^2 c$$

Note : $I = \langle S \rangle$

Electromagnetic Spectrum

The orderly distribution of electromagnetic radiations according to their wavelength or frequency is called electromagnetic spectrum.

The various regions of electromagnetic spectrum are described as follows :

- **Radiowaves :** They are produced by the accelerated motion of charges in conducting wires. They are used in radio and television communication systems.
- **Microwaves :** They are produced by special vacuum tubes called klystrons, magnetrons and Gun diodes. They are used in radar systems for aircraft navigation. Microwave oven is a domestic application of these waves.
- **Infrared waves :** They are produced by hot bodies and molecules. Infrared waves are sometimes called as **heat waves.** Infrared lamps are used in physical therapy. Infrared radiation also plays an important role in maintaining the earth's warmth or average temperature through the greenhouse effect. Infrared detectors are used in Earth satellites. Electronic devices (for example semiconductor light emitting diodes) also emit infrared and are widely used in the remote switches of household electronic systems such as TV sets, video recorders and hi-fi systems.
- **Visible rays :** It is the most familiar form of electromagnetic waves. It is the part of the spectrum that is detected by the human eye. It runs from about 4×10^{14} Hz to about 7×10^{14} Hz or a wavelength range of about 700-400 nm. Visible light emitted or reflected from objects around us provides us information about the world. Our eyes are sensitive to this range of wavelengths. Different animals are sensitive to different range of wavelengths.
- **Ultraviolet rays :** Ultraviolet (UV) radiation is produced by special lamps and very hot bodies. The sun is an important source of ultraviolet light. UV light in large quantities has harmful effects on humans. Exposure to UV radiation induces the production of more melanin, causing tanning of the skin. UV lamps are used to kill germs in water purifiers.
- **X-rays :** X-rays are used as a diagnostic tool in medicine and as a treatment for certain forms of cancer.
- **Gamma rays :** They lie in the upper frequency range of the electromagnetic spectrum. They are produced in nuclear reactions and are also emitted by radioactive nuclei. They are used in medicine to destroy cancer cell.

QUESTIONS FOR PRACTICE

1. A helium nucleus makes a full rotation in a circle of radius 0.8 m in 2 s. The value of the magnetic field induction B in tesla at the centre of circle will be

   (a) $2 \times 10^{-19}\,\mu_0$ (b) $\dfrac{10^{-19}}{\mu_0}$

   (c) $10^{-19}\,\mu_0$ (d) $\dfrac{2\times10^{-19}}{\mu_0}$

2. A torque required to hold a small circular coil of 10 turns, 2×10^{-4} m^2 area and carrying 0.5 A current in the middle of a long solenoid of 10^3 turns/m carrying 3 A current, with its axis perpendicular to the axis of the solenoid, is

   (a) $12\pi \times 10^{-7}$ N m (b) $6\pi \times 10^{-7}$ N m
   (c) $4\pi \times 10^{-7}$ N m (d) $2\pi \times 10^{-7}$ N m

3. A conducting circular loop of radius a and resistance R is kept on a horizontal plane. A vertical time varying magnetic field $B = 2t$ is switched on at time $t = 0$. Then

   (a) power generated in the coil at any time t is constant
   (b) flow of charge per unit time from any section of the coil is constant
   (c) total charge passed through any section between time $t = 0$ to $t = 2$ s is $\left(\dfrac{4\pi a^2}{R}\right)$
   (d) all of the above.

4. A coil in the shape of an equilateral triangle of side l is suspended between the pole pieces of a permanent magnet, such that $\vec{B}$ is in plane of the coil. If due to a current I in the triangle, a torque $\vec{\tau}$ acts on it, the side l of the triangle is

   (a) $\dfrac{2}{\sqrt{3}}\left(\dfrac{\tau}{BI}\right)$ (b) $2\left(\dfrac{\tau}{\sqrt{3}\,BI}\right)^{1/2}$

   (c) $\dfrac{2}{\sqrt{3}}\left(\dfrac{\tau}{BI}\right)^{1/2}$ (d) $\dfrac{1}{\sqrt{3}}\left(\dfrac{\tau}{BI}\right)$

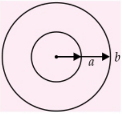
5. At a point on the right bisector of a magnetic dipole, the magnetic

   (a) potential varies as $\dfrac{1}{r^2}$
   (b) potential is zero at all points on the right bisector
   (c) field varies as r^3
   (d) field is perpendicular to the axis of dipole.

6. In an LR circuit $\nu = 50$ Hz, $L = 2$ H, $V = 5$ V, $R = 1\ \Omega$, energy stored in inductor is

   (a) 50 J (b) 25 J
   (c) 3.66×10^{-4} J (d) 6.33×10^{-5} J

7. Two concentric and coplanar circular coils have radii a and b as shown in figure. Resistance of the inner coil is R. Current in the other coil is increased from 0 to I, then the total charge circulating the inner coil is



   (a) $\dfrac{\mu_0 I ab}{2R}$ (b) $\dfrac{\mu_0 I a \pi b^2}{2ab}$

   (c) $\dfrac{\mu_0 I b}{2\pi R}$ (d) $\dfrac{\mu_0 I a^2}{2Rb}$

8. The dielectric strength of air is 3×10^6 V m^{-1}. A parallel plate capacitor has area 20 cm^2 and plate separation 0.1 mm. Find the maximum rms voltage of an ac source which can be connected.

   (a) 210 V (b) 300 V
   (c) 435 V (d) 490 V

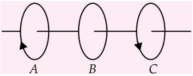
9. A galvanometer of resistance 20 Ω gives a full scale deflection when a current of 0.04 A is passed through it. It is desired to convert it into an ammeter of range 20 A. The only shunt available is 0.05 Ω. The resistance that must be connected in series with the coil of the galvanometer is

   (a) 4.95 Ω (b) 5.94 Ω
   (c) 9.45 Ω (d) 12.62 Ω

10. At a given plane on the earth's surface, the horizontal component of earth's magnetic field is 3×10^{-5} T and resultant magnetic field is 6×10^{-5} T. Angle of dip at this place is

    (a) 30° (b) 40° (c) 50° (d) 60°

11. Three identical coils, A, B and C are placed with their planes parallel to one another. Coils A and C carry currents as shown in figure. Coils B and C are fixed in the position and coil A is moved towards B. Then, current induced in B is in

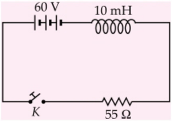


(a) clockwise direction
(b) anticlockwise direction
(c) no current is induced in B
(d) current is induced only when both coils move.

12. A coil has an inductance of 0.7 H and is joined in series with a resistance of 220 Ω. When an alternating e.m.f. of 220 V at 50 cycles per second, is applied to it, then wattless component of current in the circuit is
(a) 7 A (b) 5 A
(c) 0.7 A (d) 0.5 A

13. A long straight wire carrying a current of 30 A is placed in an external uniform magnetic field of induction 4×10^{-4} T. The magnetic field is acting parallel to the direction of current. The magnitude of the resultant magnetic induction in tesla at a point 2 cm away from the wire is
($\mu_0 = 4\pi \times 10^{-7}$ H m^{-1})
(a) 10^{-4} (b) 3×10^{-4}
(c) 5×10^{-4} (d) 6×10^{-4}

14. The rate of change of current is 500 A s^{-1} at the instant the key is pressed in the circuit shown in figure. The current through the circuit is



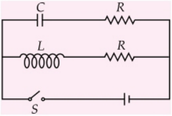
(a) 2 A (b) 1 A
(c) 0.5 A (d) 1.5 A

15. In an electromagnetic wave, the electric and magnetic fields are 100 V m^{-1} and 0.265 A m^{-1}. The maximum energy flow will be
(a) 79 W m^{-2} (b) 13.2 W m^{-2}
(c) 53.0 W m^{-2} (d) 26.5 W m^{-2}

16. There are three wavelengths : 10^{-8} m, 10^{-2} m, 10^{8} m. Their respective names are
(a) visible rays, γ-rays, ultraviolet rays
(b) ultraviolet , microwaves, radiowaves
(c) X-rays, visible rays, radiowaves
(d) radiowaves, X-rays, microwaves.

17. In the circuit shown in the figure, the switch S is closed at time $t = 0$.

$\left(\text{Given, } R = \sqrt{\frac{L}{C}}\right)$



The current through the capacitor and inductor will be equal at time t equals
(a) RC (b) $RC \ln 2$
(c) $\frac{1}{RC \ln 2}$ (d) LR

Directions : Question number 18 and 19 contain, a statement of assertion which is followed by a statement of reason.
Mark the correct choice as :
(a) if both assertion and reason are true and reason is the correct explanation of the assertion.
(b) if both assertion and reason are true but reason is not the correct explanation of the assertion.
(c) if assertion is true, but reason is false.
(d) both assertion and reason are false statements.

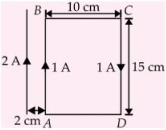
18. **Assertion :** The presence of large magnetic flux through a coil maintains a current in the coil, if the circuit is continuous.

Reason : Only a change in magnetic flux will maintain an induced current in the coil.

19. **Assertion :** Magnetic susceptibility is a pure number.

Reason : The value of magnetic susceptibility for vacuum is one.

20. What is the net force on the rectangular coil shown in figure?



(a) 25×10^{-7} N towards wire
(b) 25×10^{-7} N away from wire
(c) 35×10^{-7} N towards wire
(d) 35×10^{-7} N away from wire

21. Two wires of same length are shaped into a square and a circle. If they carry same current, ratio of their magnetic moments is
(a) $2:\pi$ (b) $\pi:2$ (c) $\pi:4$ (d) $4:\pi$

22. If a resistance of 100 Ω, an inductance of 0.5 H and a capacitance of 10×10^{-6} F are connected in series through 50 Hz ac supply, the impedance will be
(a) 1.87 Ω (b) 101.3 Ω
(c) 18.7 Ω (d) 189.7 Ω

23. One requires 11 eV of energy to dissociate a carbon monoxide molecule into carbon and oxygen atoms. The minimum frequency of the appropriate electromagnetic radiation to achieve the dissociation lies in
(a) visible region (b) infrared region
(c) ultraviolet region (d) microwave region.

24. A solenoid of 1.5 m length and 4.0 cm diameter possesses 10 turn per cm. A current of 5 A is flowing through it. The magnetic induction at axis inside the solenoid is
(a) $2\pi \times 10^{-3}$ T (b) $2\pi \times 10^{-5}$ T
(c) $2\pi \times 10^{-2}$ G (d) $2\pi \times 10^{-5}$ G

25. A wire of length l is bent in the form of a circular coil of some turns. A current I flows through the coil. The coil is placed in a uniform magnetic field B. The maximum torque on the coil can be
(a) $\frac{IBl^2}{2\pi}$ (b) $\frac{IBl^2}{4\pi}$ (c) $\frac{IBl^2}{\pi}$ (d) $\frac{2IBl^2}{\pi}$

26. A loop of area 0.5 m^2 is placed in a magnetic field of strength 2 T in direction making an angle of 30° with the field. The magnetic flux linked with the loop will be
(a) $\frac{1}{2}$ Wb (b) $\sqrt{\frac{3}{2}}$ Wb
(c) 2 Wb (d) $\frac{\sqrt{3}}{2}$ Wb

27. A transformer is used to light 140 W, 24 V lamp from 240 V ac mains. The current in the mains is 0.7 A. The efficiency of transformer is nearest to
(a) 90% (b) 80% (c) 70% (d) 60%

28. The magnetic flux through a coreless solenoid carrying current I is 5×10^{-6} Wb. If the length of solenoid is 25 cm, its magnetic moment is equal to
(a) 1 A m^2 (b) 10 A m^2
(c) 12.5 A m^2 (d) 125 A m^2

29. A circular coil of radius 0.1 m has 80 turns of wire. If the magnetic field through the coil increases from 0 to 2 T in 0.4 s and the coil is connected to a 11 Ω resistor, what is the current through the resistor during the 0.4 s?
(a) $\left(\frac{8}{7}\right)$A (b) $\left(\frac{7}{8}\right)$A (c) 8 A (d) 7 A

30. A power transformer with an 8 : 1 turn ratio has 60 Hz, 120 V across the primary, the load in the secondary is 10^4 Ω. The current in the secondary is
(a) 96 A (b) 0.96 A (c) 9.6 A (d) 96 mA

SOLUTIONS

1. **(c) :** $B = \frac{\mu_0}{4\pi}\frac{2\pi I}{r}$

Here, $I = \frac{2e}{t} = \frac{2 \times 1.6 \times 10^{-19}}{2} = 1.6 \times 10^{-19}$ A

$\therefore \quad B = \frac{\mu_0 I}{2r} = \frac{\mu_0 \times 1.6 \times 10^{-19}}{2 \times 0.8}$

$= \mu_0 \times 10^{-19}$ T

2. **(a) :** Magnetic dipole moment of circular loop is
$m = NIA = 10 \times 0.5 \times 2 \times 10^{-4} = 10^{-3}$ A m^2
Magnetic field inside the solenoid carrying current
$B = \mu_0 nI = 4\pi \times 10^{-7} \times 10^3 \times 3$
$= 12\pi \times 10^{-4}$ T
Torque, $\tau = mB\sin\theta$
$= 10^{-3} \times 12\pi \times 10^{-4} \times \sin 90°$
$= 12\pi \times 10^{-7}$ N m

3. **(d):** Here, $B = 2t$

$\therefore \quad \frac{dB}{dt} = 2$

Induced e.m.f., $|\varepsilon| = \frac{d\phi}{dt} = A\frac{dB}{dt} = 2\pi a^2$

Flow of charge per unit time through any section of the coil = induced current,

$I = \frac{\varepsilon}{R} = \frac{2\pi a^2}{R} = \text{constant.}$

Also, power generated, $P = I^2R$ = constant
Total charge passed through any section between $t = 0$ to $t = 2$ s is

$q = It = \left(\frac{2\pi a^2}{R}\right)(2-0) = \frac{4\pi a^2}{R}$

4. **(b):** Normal to the plane of the coil will be perpendicular to the field $\vec{B}$.
$\therefore \quad \tau = IBA\sin 90° = IBA$
Area of equilateral triangle,

$$A = \frac{1}{2} \times \text{Base} \times \text{Height} = \frac{1}{2} \times l \times l \sin 60^\circ = \frac{\sqrt{3}}{4} l^2$$

$$\therefore \quad \tau = IB \times \frac{\sqrt{3} l^2}{4} \text{ or } l = 2\left(\frac{\tau}{\sqrt{3} BI}\right)^{1/2}$$

5. **(b):** Magnetic potential at any point is the amount of work done in bringing a unit north pole from infinity to that point. At any point on the right bisector, the potentials due to the two poles are equal and opposite.

6. **(d):** Impedance of the *LR* circuit

$$Z = \sqrt{R^2 + 4\pi^2 \nu^2 L^2}$$

$$= \sqrt{1^2 + 4\pi^2 (50)^2 \times 2^2} = \sqrt{394385} \approx 628$$

$$I = \frac{V}{Z} = \frac{5}{628} \text{ A}$$

Energy stored in the inductor,

$$U = \frac{1}{2} L I^2 = \frac{1}{2} \times 2 \times \left(\frac{5}{628}\right)^2 \text{ J} = 6.33 \times 10^{-5} \text{ J}$$

7. **(d):** Initial flux linked with inner coil when $I = 0$ is zero. Final flux linked with inner coil when $I = I$ is $\left(\frac{\mu_0 I}{2\pi b}\right)\pi a^2$.

$\therefore$ Change in flux, $d\phi = \left(\frac{\mu_0 I}{2\pi b}\right)\pi a^2$

As $dq = \frac{d\phi}{R}$

$\therefore$ Total charge circulating the inner coil

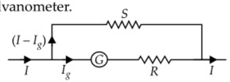
$$= \left(\frac{\mu_0 I}{2\pi b}\right)\frac{\pi a^2}{R} = \frac{\mu_0 I a^2}{2Rb}$$

8. **(a):** Electric field, $E = \frac{V}{d}$

$\therefore \quad V = Ed = 3 \times 10^6 \times (10^{-4}) = 300$ V

$$V_{\text{rms}} = \frac{V_0}{\sqrt{2}} = 300(0.707) \approx 210 \text{ V}$$

9. **(a):** Let R be resistance connected in series with the galvanometer.



From figure,

$$\frac{I_g}{I - I_g} = \frac{S}{G + R}$$

$$R = S\left(\frac{I}{I_g} - 1\right) - G$$

Substituting the given values, we get

$$R = 0.05\left(\frac{20}{0.04} - 1\right) - 20$$

$R = 4.95\ \Omega$

10. **(d):** Horizontal component of earth's magnetic field, $B_H = B_e \cos\theta$

$$\cos\theta = \frac{B_H}{B_e} = \frac{3 \times 10^{-5}}{6 \times 10^{-5}} = \frac{1}{2}$$

$\therefore \quad \theta = 60^\circ$

11. **(b):** As coil *A* is moved closer to *B*, field due to *A* intercepting *B* is increasing. Induced current in *B* must oppose this increase. Hence the current in *B* must be anticlockwise.

12. **(c):** Here, $X_L = \omega L = 2\pi\nu L$
$= 2\pi \times 50 \times 0.7 = 220\ \Omega$

$R = 220\ \Omega$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{220^2 + 220^2} = 220\sqrt{2}\ \Omega.$$

$$I_v = \frac{\varepsilon_v}{Z} = \frac{220}{220\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}} = 0.707 \text{ A}$$

Through *L*, the current is wattless.

13. **(c):** $B_1 = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi \times 10^{-7}) \times 30}{2\pi \times 0.02} = 3 \times 10^{-4}$

The direction of B_1 will be perpendicular to B_2 $(= 4 \times 10^{-4}$ T)

Hence, resultant magnetic field at given point is

$$B = \sqrt{B_1^2 + B_2^2}$$

$= [(3 \times 10^{-4})^2 + (4 \times 10^{-4})^2]^{1/2} = 5 \times 10^{-4}$ T.

14. **(b):** Induced e.m.f. across *L* is

$$\varepsilon = L\frac{dI}{dt} = 10^{-2} \times 500 = 5\text{V}$$

$\therefore$ Potential difference across *R* is,

$V = 60 - 5 = 55$ V

$$\therefore \quad I = \frac{V}{R} = \frac{55}{55} = 1 \text{ A}$$

15. **(d):** Maximum energy flow in an electromagnetic wave,
$S = E_0 \times B_0 = 100 \times 0.265 = 26.5$ W m^{-2}

16. **(b):** $\lambda = 10^{-8}$ m $\rightarrow$ ultraviolet rays,
$\lambda = 10^{-2}$ m $\rightarrow$ microwaves,
$\lambda = 10^{8}$ m $\rightarrow$ radiowaves.

17. **(b):** Growth of current in *RC* circuit
$I_C = I_0 e^{-t/RC}$)

Growth of current in LR circuit

$$I_L = I_0(1 - e^{-Rt/L}) \qquad \left[\because R = \sqrt{\frac{L}{C}} \text{ or } \frac{R}{L} = \frac{1}{RC}\right]$$
$$= I_0(1 - e^{-t/RC})$$

But $I_C = I_L$

$\therefore \quad I_0 e^{-t/RC} = I_0(1 - e^{-t/RC})$

or $\quad 2e^{-t/RC} = 1$

or $\quad e^{t/RC} = 2$

or $\quad \dfrac{t}{RC} = \ln 2$

or $\quad t = RC \ln 2.$

18. (d): If there is no change in the magnetic flux linked with the coil, there is no induced current. The current induced in a coil is directly proportional to the rate of change of magnetic flux linked with the coil.

19. (c) : $\chi_m = \dfrac{\text{Intensity of magnetisation}}{\text{Magnetising field intensity}} = \dfrac{M}{H}$

As both M and H have same units (A m^{-1}), so χ_m is a pure number. But $\chi_m = 0$ for vacuum because there can be no magnetisation in vacuum.

20. (a) : $F_{AB} = k_m\left(\dfrac{2I_1I_2}{r}\right)l = 30 \times 10^{-7}$ N (attractive)

(as $l = 15$ cm $= 15 \times 10^{-2}$ m, $r = 2$ cm $= 2 \times 10^{-2}$ m)

$F_{CD} = k_m\left(\dfrac{2I_1I_2}{r'}\right)l = 5 \times 10^{-7}$ N (repulsive)

(as $r' = (10 + 2)$ cm $= 12 \times 10^{-2}$ m)

$F_{\text{net}} = F_{AB} - F_{CD} = 25 \times 10^{-7}$ N towards wire

(as $F_{BC} = F_{AD} = 0$)

21. (c) : Let l be the length of each wire.

For circular loop,

$$m_1 = IA = I(\pi r^2) = I\pi\left(\frac{l}{2\pi}\right)^2 = \frac{Il^2}{4\pi} \qquad (\because 2\pi r = l)$$

For square loop, $m_2 = IA = I\left(\dfrac{l}{4}\right)^2 = \dfrac{Il^2}{16}$

(as side of the square $= l/4$)

Thus, $\dfrac{m_2}{m_1} = \dfrac{Il^2/16}{Il^2/4\pi} = \dfrac{\pi}{4}$

22. (d): As $X_L = 2\pi\nu L = 2\pi(50)(0.5) = 157.1\ \Omega$,

$$X_C = \frac{1}{2\pi\nu C} = \frac{1}{2\pi(50)(10^{-5})}\ \Omega = 318.3\ \Omega,$$

$|X_L - X_C| = 161.2\ \Omega$,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$
$$= \sqrt{(100)^2 + (161.2)^2}\ \Omega = 189.7\ \Omega$$

23. (c) : As $E = h\nu$,

$$\nu = \frac{E}{h} = \frac{11\text{ eV}}{4.14 \times 10^{-15}\text{ eV s}} = 2.7 \times 10^{15}\text{ Hz}$$

Frequency range of UV is 10^{14} Hz to 10^{17} Hz

24. (a) : $B = \mu_0 nI$

Here $n = 10$ turns $cm^{-1} = 1000$ turns m^{-1}

$I = 5$ A, $\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1}.

$B = 4\pi \times 10^{-7} \times 1000 \times 5 = 2\pi \times 10^{-3}$ T

25. (b): Let r be the radius of the coil and n be the number of turns formed. Then

$$l = 2\pi rn \text{ or } r = \frac{l}{2\pi n} \qquad \text{... (i)}$$

Maximum torque, $\tau_{\max} = BnIA = BnI\pi r^2$

$$= BnI\pi \times \frac{l^2}{4\pi^2 n^2} = \frac{IBl^2}{4\pi n}$$

Torque will be maximum if $n = 1$

$$\tau_{\max} = \frac{BIl^2}{4\pi}$$

26. (d): Here, $A = 0.5\ m^2$, $B = 2$ T, $\theta = 30°$

$$\phi = BA\cos\theta = 2 \times 0.5\cos 30° = \frac{\sqrt{3}}{2}\text{ Wb}$$

27. (b): $P_i = 240 \times 0.7 = 168$ W, $P_0 = 140$ W

$$\eta = \frac{P_0}{P_i} \times 100 = \frac{140}{168} \times 100 \approx 80\%$$

28. (a) : Magnetic flux, $\phi = BA = \left(\mu_0 \dfrac{N}{l} I\right)A$

Magnetic moment $= NIA = \dfrac{\phi l}{\mu_0}$

$$= \frac{5 \times 10^{-6} \times 0.25}{4\pi \times 10^{-7}} \approx 1\text{ A m}^2$$

29. (a) : Here, $r = 0.1$ m, $N = 80$,

$$\frac{dB}{dt} = \frac{2 - 0}{0.4} = 5\text{ T s}^{-1},\ R = 11\ \Omega$$

$$I = \frac{\varepsilon}{R} = \frac{NA}{R}\frac{dB}{dt} = \frac{N(\pi r^2)}{R} \times \frac{dB}{dt}$$

$$= 80 \times \frac{22}{7}\frac{(0.1)^2}{11} \times 5 = \frac{8}{7}\text{ A}$$

30. (d): Transformation ratio $\dfrac{n_s}{n_p} = \dfrac{8}{1}$

$V_p = 120$ V, $R_s = 10^4\ \Omega$

$$V_s = V_p \times \frac{n_s}{n_p} = 120 \times 8 = 960\text{ V}$$

$$I_s = \frac{V_s}{R_s} = \frac{960}{10^4} = 96 \times 10^{-3}\text{ A} = 96\text{ mA}$$

EXAMINER'S MIND

NCERT Class XI

The questions given in this column have been prepared strictly on the basis of NCERT Physics for Class XI. This year JEE (Main and Advanced)/NEET/AIIMS/other PMTs have drawn their papers heavily from NCERT books.

SECTION - 1

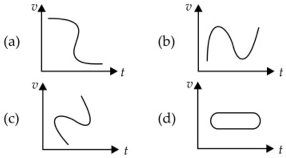
Only One Option Correct Type

This section contains 15 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which ONLY ONE is correct.

1. A wire has a mass of (0.3 ± 0.003) g, radius (0.5 ± 0.005) mm and length (6 ± 0.06) cm. The maximum percentage error in the measurement of density is

(a) 1 % (b) 2 % (c) 3 % (d) 4 %

2. Which of the following velocity-time graphs shows a realistic situation for a body in motion?



3. A point initially at rest moves along x-axis. Its acceleration varies with time as $a = (6t + 5)$ m s^{-2}. If it starts from origin, the distance covered in 2 seconds is

(a) 20 m (b) 18 m (c) 16 m (d) 25 m

4. A ball is thrown from the ground with a velocity of $20\sqrt{3}$ m s^{-1} making an angle of 60° with the horizontal. The ball will be at a height of 40 m from the ground after a time t equal to (Take $g = 10$ m s^{-2})

(a) $\sqrt{2}$ s (b) $\sqrt{3}$ s (c) 2 s (d) $2\sqrt{3}$ s

5. A body is moving up an inclined plane of angle θ with an initial kinetic energy K. The coefficient of friction between the plane and the body is μ. The work done against friction before the body comes to rest is

(a) $\dfrac{\mu\cos\theta}{K\cos\theta + \sin\theta}$ (b) $\mu K\cos\theta$

(c) $\dfrac{\mu K\cos\theta}{\mu\cos\theta - \sin\theta}$ (d) $\dfrac{\mu K\cos\theta}{\mu\cos\theta + \sin\theta}$

6. A ball impinges directly on a similar ball at rest. The first ball is brought to rest by the impact. If half of the kinetic energy is lost by impact, the value of coefficient of restitution is

(a) $\dfrac{1}{2\sqrt{2}}$ (b) $\dfrac{1}{\sqrt{3}}$ (c) $\dfrac{1}{\sqrt{2}}$ (d) $\dfrac{\sqrt{3}}{2}$

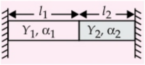
7. When a sphere rolls without slipping, the ratio of its kinetic energy of translation to its total kinetic energy is

(a) 1 : 7 (b) 1 : 2 (c) 1 : 1 (d) 5 : 7

8. The acceleration due to gravity at the poles and the equator is g_p and g_e respectively. If the earth is a sphere of radius R and rotating about its axis with angular speed ω, then $g_p - g_e$ is given by

(a) $\dfrac{\omega^2}{R}$ (b) $\dfrac{\omega^2}{R^2}$ (c) $\omega^2 R^2$ (d) $\omega^2 R$

9. Two rods of different materials having the same area of cross-section A, are placed between two massive walls as shown in figure. The first rod has a length l_1, coefficient of linear expansion α_1 and Young's modulus Y_1. The corresponding quantities for the second rod are l_2, α_2 and Y_2. The temperature of both ends is now raised by T degrees. The force with which the rods act on each other at higher temperature is given by



(a) $F = \dfrac{T(l_1\alpha_1^2 + l_2\alpha_2^2)A}{\left(\dfrac{l_1}{Y_1} + \dfrac{l_2}{Y_2}\right)}$

(b) $F = \dfrac{T(l_1\alpha_1 + l_2\alpha_2)A}{\left(\dfrac{l_1}{Y_1} + \dfrac{l_2}{Y_2}\right)}$

(c) $F = \dfrac{T\left(\dfrac{l_1}{Y_1} + \dfrac{l_2}{Y_2}\right)A}{(l_1\alpha_1 + l_2\alpha_2)}$ (d) $F = \dfrac{T\left(\dfrac{l_1}{\alpha_1} + \dfrac{l_2}{\alpha_2}\right)A}{(l_1Y_1 + l_2Y_2)}$

10. A soap bubble of radius r_1 is placed on another soap bubble of radius r_2 ($r_1 < r_2$). The radius R of the soapy film separating the two bubbles is

(a) $r_1 + r_2$ (b) $\sqrt{r_1^2 + r_2^2}$

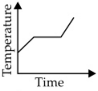
(c) $(r_1^3 + r_2^3)^{1/3}$ (d) $\dfrac{r_2 r_1}{r_2 - r_1}$

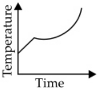
11. A solid sphere of volume V and density ρ floats at the interface of two immiscible liquids of densities ρ_1 and ρ_2 respectively. If $\rho_1 < \rho < \rho_2$, then the ratio of volume of the parts of the sphere in upper and lower liquid is

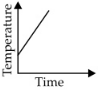
(a) $\dfrac{\rho - \rho_1}{\rho_2 - \rho}$ (b) $\dfrac{\rho_2 - \rho}{\rho - \rho_1}$

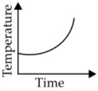
(c) $\dfrac{\rho + \rho_1}{\rho + \rho_2}$ (d) $\dfrac{\rho + \rho_2}{\rho + \rho_1}$

12. Liquid oxygen at 50 K is heated to 300 K at constant pressure of 1 atm. The rate of heating is constant. Which one of the following graphs represents the variation of temperature with time?

(a)


(b)


(c)


(d)


13. When the tension in a metal wire is T_1, its length is l_1. When the tension is T_2, its length is l_2. The natural length of wire is

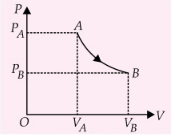
(a) $\dfrac{T_2}{T_1}(l_1 + l_2)$ (b) $T_1 l_1 + T_2 l_2$

(c) $\dfrac{l_1 T_2 - l_2 T_1}{T_2 - T_1}$ (d) $\dfrac{l_1 T_2 + l_2 T_1}{T_2 + T_1}$

14. A body cools from 50°C to 49°C in 5 s. How long will it take to cool from 40°C to 39°C ? Assume temperature of surroundings to be 30°C and Newton's law of cooling is valid.

(a) 2.5 s (b) 10 s
(c) 20 s (d) 5 s

15. 5 moles of an ideal gas is carried by a quasi-static isothermal process at 500 K to twice its volume as shown in figure.



Find the work done by the gas along the path AB and the pressure ratio $\dfrac{P_B}{P_A}$.

(Given $R = 8.31$ J mol^{-1} K^{-1}, log2 = 0.3010)

(a) 14401 J, 2 (b) 3428 J, 2

(c) 14401 J, $\dfrac{1}{2}$ (d) 3428 J, $\dfrac{1}{2}$

SECTION - 2

One or More Options Correct Type

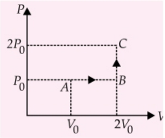
This section contains 10 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which ONE or MORE are correct.

16. The motion which is not simple harmonic is
(a) vertical oscillations of a spring
(b) motion of simple pendulum
(c) motion of a planet around the sun
(d) oscillation of liquid column in a U-tube

17. A simple pendulum of length l is set into motion such that the bob moves along a horizontal circular path, and the string makes a constant angle with the vertical. If t is the time period of motion and T is tension in the string, then
(a) $T \propto t^{-2}$ (b) $T \propto t^2$ (c) $T \propto l$ (d) $T \propto l^{-1}$

18. Two bodies A and B have thermal emissivities of 0.01 and 0.81 respectively. The outer surface areas of the two bodies are equal and emit total radiant power at same rate. The wavelength λ_B corresponding to maximum spectral radiancy in the radiation from B is shifted from the wavelength corresponding to maximum spectral radiancy in the radiation from A by 1 μm. If the temperature of A is 5802 K, then
(a) the temperature of B is 1934 K.
(b) $\lambda_B = 1.5$ μm.
(c) the temperature of B is 1160 K.
(d) $\lambda_B = 2.5$ μm.

19. One mole of an ideal monoatomic gas is taken from A to C along the path ABC shown in figure. The temperature of the gas at A is T_0. For the process ABC (where R is gas constant)



(a) heat absorbed by the gas is $\frac{11}{12}RT_0$.

(b) heat absorbed by the gas is $\frac{11}{2}RT_0$.

(c) work done by the gas is RT_0.

(d) change in internal energy of gas is $\frac{9}{2}RT_0$.

20. The potential energy of a particle of mass 0.1 kg moving along the x-axis is given by $U = 5x(x - 4)$ J where x is in metres. Which of the following is/are correct statement(s)?

(a) The particle is acted upon by a constant force.
(b) The particle executes SHM.
(c) The speed of the particle is maximum at $x = 2$ m.
(d) The period of oscillation of particle is $\pi/5$ s.

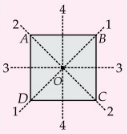
21. A wave is represented by the equation

$$y = A\sin\left(10\pi x + 15\pi t + \frac{\pi}{3}\right)$$

where x is in metres and t is in seconds. The expression represents

(a) a wave travelling in the positive X direction with a velocity of 1.5 m s^{-1}.
(b) a wave travelling in the negative X direction with a velocity of 1.5 m s^{-1}.
(c) a wave travelling in the negative X direction with a wavelength of 0.2 m.
(d) a wave travelling in the positive X direction with a wavelength of 0.2 m.

22. The moment of inertia of a thin square plate $ABCD$ of uniform thickness about an axis passing through the centre O and perpendicular to the plate is



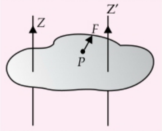
(a) $I_1 + I_2$ (b) $I_3 + I_4$
(c) $I_1 + I_3$ (d) $I_1 + I_2 + I_3 + I_4$

(Where I_1, I_2, I_3 and I_4 are respectively the moments of inertia about axes 1, 2, 3 and 4 which are in the plane of the plate).

23. A point mass of 1 kg collides elastically with a stationary point mass of 5 kg. After their collision, the 1 kg mass reverses its direction and moves with a speed of 2 m s^{-1}. Which of the following statement(s) is (are) correct for the system of these two masses?

(a) Total momentum of the system is 3 kg m s^{-1}.
(b) Momentum of 5 kg mass after collision is 4 kg m s^{-1}.
(c) Kinetic energy of the centre of mass is 0.75 J.
(d) Total kinetic energy of the system is 4 J.

24. Given figure shows a lamina in XY-plane. Two axes Z and Z' pass perpendicular to its plane. A force $\vec{F}$ acts in the plane of lamina at point P as shown. Which of the following is (are) true? (The point P is closer to Z'-axis than the Z-axis.)



(a) Torque $\vec{\tau}$ caused by $\vec{F}$ about Z-axis is along $-\hat{k}$
(b) Torque $\vec{\tau}'$ caused by $\vec{F}$ about Z'-axis is along $-\hat{k}$
(c) Torque $\vec{\tau}$ caused by $\vec{F}$ about Z-axis is greater in magnitude than that about Z'-axis.
(d) Total torque is given by $\vec{\tau}_{net} = \vec{\tau} + \vec{\tau}'$.

25. When a wire is stretched to double its length

(a) its radius is halved.
(b) strain is unity.
(c) stress is equal to Young's modulus.
(d) Young's modulus is equal to twice the elastic energy per unit volume.

SECTION -3

Paragraph Type

This section contains 2 paragraphs describing theory, experiment, data etc. Two questions related to the paragraph with two questions paragraph. Each question of paragraph has only one correct answer among the four choices (a), (b), (c) and (d).

Paragraph for Questions 26 and 27

A rocket is fired vertically upwards with a speed of v(= 5 km s^{-1}) from the surface of earth. It goes up

to a height h before returning to earth. At height h a body is thrown from the rocket with speed v_0 in such a way so that the body becomes a satellite of earth. Let the mass of the earth, $M = 6 \times 10^{24}$ kg, mean radius of the earth in a line, $R = 6.4 \times 10^6$ m, $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}, $g = 9.8$ m s^{-2}

26. The value of h is
(a) 1.5×10^5 m (b) 3.2×10^5 m
(c) 3.2×10^6 m (d) 1.6×10^6 m

27. The value of v_0 (orbital velocity) of satellite is
(a) 6.7 km s^{-1} (b) 7. 1 km s^{-1}
(c) 7.8 km s^{-1} (d) 8.2 km s^{-1}

Paragraph for Questions 28 and 29

According to Doppler's effect in sound, the apparent frequency (υ') of sound heard by a listener when the relative motion between the listener and the source is given,

$$\upsilon' = \left(\frac{v - v_L}{v - v_S}\right)\upsilon$$

where υ is actual frequency of sound emitted by the source, v is velocity of sound in air, v_L is velocity of listener and v_S is velocity of source. The velocities along SL are taken as positive and velocities along LS are taken as negative.

28. A tuning fork producing sound of frequency 256 Hz is moving towards a listener with a velocity of 20 m s^{-1}. If velocity of sound in air is 340 m s^{-1}, the apparent frequency of sound heard is
(a) 272 Hz (b) 256 Hz (c) 300 Hz (d) 340 Hz

29. In the above case, if the fork moves at right angles to the line SL, the apparent frequency of sound heard would be
(a) zero (b) 256 Hz (c) 272 Hz (d) 240 Hz

SECTION - 4

Matching List Type

This section contains 2 multiple choice questions. Questions has matching lists. The codes for the lists have choices (a), (b), (c) and (d) out of which ONLY ONE is correct.

30. Match the quantities given in List I with the horizontal range given in List II for a particle projected with velocity u.

	List I		List II
P.	Its horizontal range is thrice the greatest height attained	1.	$\frac{4}{5}\frac{u^2}{g}$
Q.	Its maximum height is thrice the horizontal range attained	2.	$\frac{24}{145}\frac{u^2}{g}$
R.	Its horizontal range is twice the greatest height attained	3.	$\frac{16}{65}\frac{u^2}{g}$
S.	Its maximum height is twice the horizontal range attained	4.	$\frac{24}{25}\frac{u^2}{g}$

Codes

	P	Q	R	S
(a)	1	2	3	4
(b)	2	4	1	3
(c)	4	2	1	3
(d)	4	2	3	1

31. A ball thrown up is caught by the thrower after 4 seconds. Use $g = 9.8$ m s^{-1}.
Match the quantities given in List I with the horizontal range given in List II.

	List I		List II
P.	The height of ball after 2 seconds	1.	14.7 m
Q.	The height of ball after 3 seconds	2.	19.6 m
R.	The speed of the ball after 3 seconds	3.	9.8 m s^{-1}
S.	The speed of ball after 4 seconds	4.	19.6 m s^{-1}

Codes

	P	Q	R	S
(a)	1	2	3	4
(b)	2	1	3	4
(c)	1	2	4	3
(d)	2	1	4	3

SECTION - 5

Assertion-Reason Type

This section contains 4 questions. Read the two statements in the following questions. Of the four choices given, choose the one that best describes the two statements..
(a) Statement-I is true, Statement-II is true; Statement-I is a correct explanation of Statement-I.
(b) Statement-I is true, Statement-II is true; Statement-II is not a correct explanation of Statement-I.
(c) Statement-I is true, Statement-II is false.
(d) Statement-I is false, Statement-II is true.

32. **Statement-I :** In simple harmonic motion the velocity is maximum when the acceleration is minimum.
Statement-II : Displacement and velocity of SHM differ in phase by $\frac{\pi}{2}$.

33. **Statement-I :** Coefficient of adiabatic elasticity of air is greater than the coefficient of isothermal elasticity.
Statement-II : Heat is exchanged freely in a isothermal change, but not in an adiabatic change.

34. **Statement-I :** A dam for water reservoir is built thicker at bottom than at the top.
Statement-II : Pressure of water is very large at the bottom.

35. **Statement-I :** Smaller drops of liquid resist deforming forces better than the larger drops.
Statement-II : Excess pressure inside a drop is directly proportional to its surface area.

SECTION - 6
Integer Value Type

This section contains 5 questions. The answer to each question is a single digit integer, ranging from 0 to 9 (both inclusive).

36. Two exactly identical rain drops falling with terminal velocity of $2^{1/3}$ m s^{-1} coalesce to form a bigger drop. Find the new terminal velocity of the bigger drop.

37. A material has Poisson's ratio 0.5. If an uniform rod of it suffers a longitudinal strain of 2×10^{-3}, what is the percentage increase in volume?

38. A particle executes SHM of period 8 s. After what time of its passing through the mean position, will the energy be half kinetic and half potential?

39. A pump motor is used to deliver water at a certain rate from a given pipe. To increase the rate of water flow two-times from the same pipe, by how much power should be increased?

40. A hoop of radius 2 m weighs 100 kg. It rolls along a horizontal floor so that its centre of mass has a speed of 20 cm s^{-1}. How much work has to be done to stop it?

SOLUTIONS

1. **(d):** Since density $\rho = \dfrac{m}{\pi r^2 l}$

$\therefore$ Maximum percentage error in density,

$$\therefore \quad \left(\frac{\Delta\rho}{\rho}\right)\times 100 = \left(\frac{\Delta m}{m} + \frac{2\Delta r}{r} + \frac{\Delta l}{l}\right)\times 100$$

$$= \left(\frac{0.003}{0.3} + 2\times\frac{0.005}{0.5} + \frac{0.06}{6}\right)\times 100$$

$$= (0.01 + 0.02 + 0.01) \times 100 = 4\%$$

2. **(b):** In option (a), (c) and (d), we can find from graph that at a single time there is more than one velocity, which is not possible practically. Hence, v–t graph in option (b) shows a realistic situation for a body in motion.

3. **(b):** Given, $a = \dfrac{dv}{dt} = 6t + 5$ ...(i)

Integrating equation (i), $\int_0^v dv = \int_0^t (6t+5)dt$

or $\quad v = \dfrac{6t^2}{2} + 5t$

As $v = \dfrac{ds}{dt}$, or $ds = \left(\dfrac{6t^2}{2} + 5t\right)dt$ ...(ii)

Integrating equation (ii), $\int_0^s ds = \int_0^t \left(\dfrac{6t^2}{2} + 5t\right)dt$

or $\quad s = 3\dfrac{t^3}{3} + \dfrac{5t^2}{2} = t^3 + \dfrac{5}{2}t^2$

When $t = 2$ s, $s = 2^3 + \dfrac{5\times 2^2}{2} = 18$ m

4. **(c) :** As in projectile motion,

$s = u\sin\theta\ t - \dfrac{1}{2}gt^2,$ ...(i)

Here, $u = 20\sqrt{3}$ m s^{-1}

$g = 10$ m s^{-2} and $\sin\theta = \sin 60° = \dfrac{\sqrt{3}}{2}$

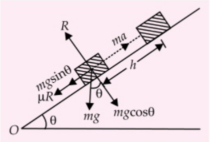
Using (i) we get

$$40 = 20\sqrt{3}\times\frac{\sqrt{3}}{2}t - \frac{1}{2}\times 10\times t^2$$

or $\quad 5t^2 - 30t + 40 = 0$ or $t^2 - 6t + 8 = 0$

$\Rightarrow \quad t = 2$ s or 4 s. Hence, the minimum time $t = 2$ s

5. **(d):** Let the distance travelled by the body on the inclined plane be h.
From the free body diagram shown in figure



$R = mg\cos\theta$ ... (i)

$-ma = \mu R + mg\sin\theta$ ... (ii)

From equations (i) and (ii)

$-ma = \mu mg\cos\theta + mg\sin\theta$

or $\quad a = -g(\mu\cos\theta + \sin\theta)$ ...(iii)

Since, $v^2 - u^2 = 2ah$

$\therefore \quad u^2 = 2ah \qquad (\because v = 0)$

and kinetic energy, $K = \frac{1}{2}mu^2$ or $\frac{u^2}{2} = \frac{K}{m}$

$\therefore \quad ah = \frac{v^2}{2} = \frac{K}{m}$ ... (iv)

Work done, $W = Fs = mg(h \sin\theta)$ ... (v)

From equations (iii) and (iv)

$$W = \frac{-mah \sin\theta}{(\mu\cos\theta + \sin\theta)} = \frac{K\sin\theta}{\mu\cos\theta + \sin\theta}$$

But $\mu = \tan\theta$

or $\sin\theta = \mu \cos\theta$

$$\therefore \quad W = \frac{K\mu\cos\theta}{\mu\cos\theta + \sin\theta}$$

6. **(c) :** Let u_1 and v_1 be the initial and final velocities of ball 1 and u_2 and v_2 be the similar quantities for ball 2. Here, $u_2 = 0$ and $v_1 = 0$.

$\therefore$ initial KE, $K_i = \frac{1}{2}mu_1^2 + \frac{1}{2}mu_2^2 = \frac{1}{2}mu_1^2$

and final KE, $K_f = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = \frac{1}{2}mv_2^2$

Loss of KE, $\Delta K = K_i - K_f = \frac{1}{2}mu_1^2 - \frac{1}{2}mv_2^2$

According to question,

$$\frac{1}{2}\left(\frac{1}{2}mu_1^2\right) = \frac{1}{2}mu_1^2 - \frac{1}{2}mv_2^2$$

($\because$ half of its KE is lost by impact)

or $u_1^2 = 2v_2^2$ or $v_2 = \frac{u_1}{\sqrt{2}}$

$\therefore$ Coefficient of restitution,

$$e = -\left(\frac{v_2 - v_1}{u_1 - u_2}\right) = \frac{v_2}{u_1} = \frac{1}{\sqrt{2}}$$

7. **(d):** The ratio of translation kinetic energy to total kinetic energy,

$$\frac{K_T}{K} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2}$$

$$= \frac{\frac{1}{2}mv^2}{\frac{1}{2}mv^2 + \frac{1}{2}\times\frac{2}{5}mr^2\times\frac{v^2}{r^2}} \quad \left(\because I = \frac{2}{5}mr^2 \text{ and } v = r\omega\right)$$

$$\therefore \quad \frac{K_T}{K} = \frac{1}{1+\frac{2}{5}} = \frac{5}{7}$$

8. **(d):** Acceleration due to gravity at a place of latitude λ due to the rotation of earth is,

$$g' = g - R\omega^2\cos^2\lambda$$

At equator, $\lambda = 0°$, $\cos 0° = 1$

$\therefore \quad g' = g_e = g - R\omega^2$ ... (i)

At poles, $\lambda = 90°$, $\cos 90° = 0$

$\therefore \quad g' = g_p = g$ ...(ii)

From equation (i) and (ii)

$$g_p - g_e = g - (g - R\omega^2) = R\omega^2$$

9. **(b):** At the higher temperature T, increase in the length of 1st rod $= l_1\alpha_1 T$,

and increase in the length of 2nd rod $= l_2\alpha_2 T$

$\therefore$ Total increase in length $= T(l_1\alpha_1 + l_2\alpha_2) = l_1\alpha_1 T + l_2\alpha_2 T$

The walls will not allow the rods to lengthen. Hence, one rod presses on the other and vice-versa producing decrease in length keeping the total length the same. Let F be the force acting on one rod due to the other then decrease in length of the 1st rod $= \frac{Fl_1}{Y_1 A}$

and decrease in length of 2nd rod $= \left(\frac{Fl_2}{Y_2 A}\right)$

$$\therefore \quad \text{Total decrease in length} = \frac{Fl_1}{Y_1 A} + \frac{Fl_2}{Y_2 A} = \frac{F}{A}\left(\frac{l_1}{Y_1} + \frac{l_2}{Y_2}\right)$$

Now, decrease in length = increase in length

$$\Rightarrow \quad T(l_1\alpha_1 + l_2\alpha_2) = \frac{F}{A}\left(\frac{l_1}{Y_1} + \frac{l_2}{Y_2}\right)$$

$$\therefore \quad F = \frac{T(l_1\alpha_1 + l_2\alpha_2)A}{\left(\frac{l_1}{Y_1} + \frac{l_2}{Y_2}\right)}$$

10. **(d):** Two soap bubbles of radii r_1 and r_2 ($r_1 < r_2$) come together to form a double bubble. Let T represents the surface tension of the liquid and P the atmospheric pressure.

Pressure inside the smaller bubble $= P + \frac{4T}{r_1}$

Pressure inside the larger bubble $= P + \frac{4T}{r_2}$

Pressure difference $= 4T\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

Now pressure on the concave side of the soapy film separating the two bubbles is greater than that on the convex side by $\frac{4T}{R}$.

$$\therefore \quad \frac{4T}{R} = 4T\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$\therefore \quad \frac{1}{R} = \frac{r_2 - r_1}{r_1 r_2} \quad \text{or} \quad R = \frac{r_1 r_2}{r_2 - r_1}$$

11. (b): V = Volume of solid sphere.

Let V_1 = Volume of the part of the sphere immersed in a liquid of density ρ_1 and V_2 = Volume of the part of the sphere immersed in liquid of density ρ_2.

According to law of floatation,

$V\rho g = V_1\rho_1 g + V_2\rho_2 g$ ... (i)

and $V = V_1 + V_2$ ... (ii)

Hence from (i) and (ii),

$V_1\rho g + V_2\rho g = V_1\rho_1 g + V_2\rho_2 g$

or $V_1(\rho - \rho_1)g = V_2(\rho_2 - \rho)g$

or $\dfrac{V_1}{V_2} = \dfrac{\rho_2 - \rho}{\rho - \rho_1}$

12. (a): Temperature of liquid oxygen will first increase in the same phase. Then, the liquid oxygen will change to gaseous phase during which temperature will remain constant. After that, temperature of oxygen in gaseous state will increase. Hence option (a) represents corresponding temperature-time graph.

13. (c): Since $Y = \dfrac{Fl}{A\Delta l}$

As Y, l and A are constants,

$\therefore \dfrac{F}{\Delta l} = \text{constant}$ or $\Delta l \propto F$

Hence, $l_1 - l \propto T_1$ ... (i)

and $l_2 - l \propto T_2$ ... (ii)

Dividing (i) by (ii), $\dfrac{l_1 - l}{l_2 - l} = \dfrac{T_1}{T_2}$

or $l_1T_2 - lT_2 = l_2T_1 - lT_1 \Rightarrow l(T_1 - T_2) = l_2T_1 - l_1T_2$

or $l = \dfrac{l_2T_1 - l_1T_2}{T_1 - T_2} \Rightarrow l = \dfrac{l_1T_2 - l_2T_1}{T_2 - T_1}$

14. (b): From Newton's law of cooling

$\dfrac{T_1 - T_2}{t} \propto \left(\dfrac{T_1 + T_2}{2} - T_0\right)$

where, T_0 = temperature of surroundings

$\therefore \dfrac{50 - 49}{t_1} \propto \left(\dfrac{50 + 49}{2} - 30\right)$

or $\dfrac{1}{t_1} = K(19.5)$ ... (i)

and $\dfrac{40 - 39}{t_2} \propto \left(\dfrac{40 + 39}{2} - 30\right)$

or $\dfrac{1}{t_2} = K(9.5)$ ... (ii)

Dividing equation (i) by (ii), we get

$\dfrac{t_2}{t_1} = \dfrac{19.5}{9.5}$

$\Rightarrow t_2 = \dfrac{19.5}{9.5} \times 5 \simeq 10 \text{ s}$

15. (c): Here, $n = 5$, $T = 500$ K, $V_B = 2V_A$, $R = 8.31 \text{ J mol}^{-1}\text{K}^{-1}$

Now, in the quasi-static isothermal process work done,

$$W_{\text{iso.}} = n \times 2.303 \times RT \log \frac{V_B}{V_A}$$

$$= 5 \times 2.303 \times 8.31 \times 500 \log \frac{2V_A}{V_A}$$

$= 5 \times 2.303 \times 8.31 \times 500 \log 2$

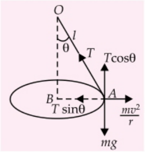
$= 5 \times 2.303 \times 8.31 \times 500 \times 0.3010 = 14401$ J

For an isothermal change, $P_AV_A = P_BV_B$

$\therefore \dfrac{P_B}{P_A} = \dfrac{V_A}{V_B} = \dfrac{V_A}{2V_A} = \dfrac{1}{2}$

16. (c): The motion of a planet around the sun is a periodic motion but not a simple harmonic motion. All other given motions are the examples of simple harmonic motion.

17. (a,c): In case of conical pendulum shown in figure



$T\cos\theta = mg$ ... (i)

and $T\sin\theta = \dfrac{mv^2}{r} = mr\omega^2 = mr\dfrac{4\pi^2}{t^2}$ ... (ii)

and $r = l\sin\theta$

or $T\sin\theta = ml\sin\theta\dfrac{4\pi^2}{t^2} = ml\dfrac{4\pi^2}{t}$

$\therefore T \propto l$ and $T \propto t^{-2}$

18. (a,b): Radiant power, $P = e\sigma AT^4$

As $P_A = P_B$ and surface areas of the two bodies are equal,

$\therefore e_AT_A^4 = e_BT_B^4$

$$T_B = \left(\frac{e_A}{e_B}\right)^{1/4} . T_A = \left(\frac{0.01}{0.81}\right)^{1/4} \times 5802 = 1934 \text{ K}$$

According to Wien's law,

$\lambda_AT_A = \lambda_BT_B$

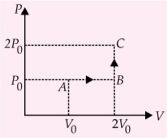
$\therefore \lambda_B = \lambda_A\dfrac{T_A}{T_B} = \dfrac{\lambda_A \times 5802}{1934} = 3\lambda_A$

Also $\lambda_B - \lambda_A = 1\ \mu m$

$$\lambda_B - \frac{\lambda_B}{3} = 1$$

or $\lambda_B = 1.5\ \mu m$

19. (b,c,d) : In the process AB, P = constant.



$\therefore \quad V \propto T.$

As $V_B = 2V_0 = 2\,V_A$,

or $T_B = 2\,T_A = 2T_0$

$\therefore \quad \Delta W_{AB} = P_0(2V_0 - V_0) = P_0V_0 = RT_0$

$$\Delta Q_{AB} = C_P\Delta T = \left(\frac{5}{2}R\right)\Delta T$$

[$\because$ For mono atomic gas $C_P = \frac{5}{2}R$]

$$= \left(\frac{5}{2}R\right)(2T_0 - T_0) = \frac{5}{2}RT_0$$

$\therefore \quad \Delta U_{AB} = \Delta Q_{AB} - \Delta W_{AB}$

$$= \frac{5}{2}RT_0 - RT_0 = \frac{3}{2}RT_0$$

In the process BC,

V = constant

$\therefore \quad P \propto T$

As $P_C = 2P_0 = 2P_B$, $\therefore \quad T_C = 2T_B = 4T_0$

$\Delta W_{BC} = 0$ ($\because$ change in volume is zero)

$\therefore \quad \Delta Q_{BC} = \Delta U_{BC} = C_v(\Delta T)$

$$= \left(\frac{3}{2}R\right)(4T_0 - 2T_0) = 3RT_0$$

$\therefore \quad \Delta W_{net} = \Delta W_{AB} + \Delta W_{BC} = RT_0 + 0 = RT_0$

$\Delta Q_{net} = \Delta Q_{AB} + \Delta Q_{BC}$

$$= \frac{5}{2}RT_0 + 3RT_0 = \frac{11}{2}RT_0$$

Now, $\Delta U_{net} = \Delta Q_{net} - \Delta W_{net}$

$$= \frac{11}{2}RT_0 - RT_0 = \frac{9}{2}RT_0$$

20. (b,c,d) : Here, $U = 5x(x-4) = (5x^2 - 20x)$ J

$\therefore \quad F = -\frac{dU}{dx} = -[10x - 20] = -(10x + 20)$ N ... (i)

As F changes with x, so F is not constant.

Since $F \propto x$ and it is directed towards mean position, hence the particle executes SHM.

In SHM, the speed is maximum at mean position where force is zero.

$\therefore \quad 0 = -10x + 20$ or $x = 2$ m

Here, $m\omega^2 = 10$

or $\omega^2 = \frac{10}{m} = \frac{10}{0.1} = 100$ or $\omega = 10$ rad s^{-1}

$\therefore \quad T = \frac{2\pi}{\omega} = \frac{2\pi}{10} = \frac{\pi}{5}$ s.

21. (b,c) : Compare the given equation with standard form

$y = A\sin\left(\frac{2\pi x}{\lambda} + \frac{2\pi t}{T} + \phi_0\right)$, we get

$$\frac{2\pi}{\lambda} = 10\pi \quad \Rightarrow \quad \lambda = \frac{2\pi}{10\pi} = 0.2\ \text{m}$$

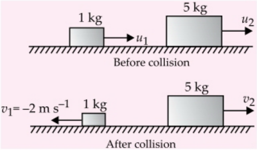
$$\frac{2\pi}{T} = 15\pi \quad \Rightarrow \quad T = \frac{2}{15}\ \text{s},$$

$$\therefore \quad v = \frac{\lambda}{T} = \frac{0.2}{2/15} = 1.5\ \text{m s}^{-1}$$

In the equation, x is positive therefore the wave is travelling along negative direction of X-axis.

22. (a,b,c)

23. (a,c) :



From law of conservation of linear momentum,

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

or $1 \times u_1 + m_2 \times 0 = 1(-2) + 5v_2$

or $u_1 = 5v_2 - 2$ ... (i)

Since the collision is elastic,

velocity of approach = velocity of separation after collision

i.e. $u_1 - u_2 = v_2 - v_1$

or $u_1 - 0 = v_2 - (-2)$

or $u_1 = v_2 + 2$ ... (ii)

From equation (i) and (ii),

$v_2 = 1$ m s^{-1} and $u_1 = 3$ m s^{-1}

(a) Total momemtum of the system

$= m_1u_1 + m_2u_2$

$= 1\text{ kg} \times 3\text{ m s}^{-1} + m_2 \times 0 = 3\text{ kg m s}^{-1}$

(b) Momentum of 5 kg after collsion $= m_2v_2$

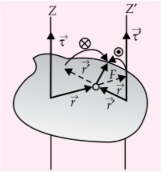
$= 5\text{ kg} \times 1\text{ m s}^{-1} = 5\text{ kg m s}^{-1}$

(c) KE of centre of mass $= \frac{1}{2}m_{\text{system}}v_{\text{cm}}^2$ ...(i)

$$\left(v_{cm} = \frac{m_1u_1 + m_2u_2}{m_1 + m_2} = \frac{1 \times 3 + m \times 0}{1 + 5} = \frac{3}{6} = 0.5\ \text{m s}^{-1}\right)$$

$$= \frac{1}{2}(1\text{ kg} + 5\text{ kg})(0.5\text{ m s}^{-1})^2 = 0.75\text{ J}$$

24. (b,c) :



(a) As $\vec{\tau}=\vec{r}\times\vec{F}$ and $\vec{r}\times\vec{F}$ is along Z-axis (*i.e.*, $\vec{\tau}$ is anticlockwise),
Hence, $\vec{\tau}$ is along $\hat{k}$ direction.

(b) As $\vec{\tau}'=\vec{r}'\times\vec{F}$ and $\vec{r}'\times\vec{F}$ is along – Z-axis (*i.e.*, $\vec{\tau}'$ is clockwise)
Hence $\vec{\tau}'$ is along $-\hat{k}$ direction

(c) Since $\vec{F}$ is farther from Z-axis, $|\vec{\tau}|>|\vec{\tau}'|$.

(d) $\vec{\tau}_{net}\neq\vec{\tau}+\vec{\tau}'$ as there is no sense in adding torques about two different axes.

25. (b,c,d) : (b) Strain $=\frac{\Delta l}{l}=\frac{l}{l}=1$

(c) $Y=\frac{\text{stress}}{\text{strain}}=\text{stress}$

(d) Elastic energy per unit volume, *i.e.*,

$$u=\left(\frac{1}{2}\right)\text{stress}\times\text{strain}$$

or $u=\frac{1}{2}\times Y\times 1$

$\therefore\ Y=2u$ $\quad$ ($\because$ stress = Y and strain = 1)

26. (d): According to law of conservation of total mechanical energy,
total energy of rocket at the surface of earth = total energy of rocket at the highest point

or $\frac{1}{2}mv^2+\left(\frac{-GMm}{R}\right)=0+\left(\frac{-GMm}{R+h}\right)$

or $\frac{v^2}{2}=\frac{GM}{R}-\frac{GM}{(R+h)}=\frac{gR^2}{R}-\frac{gR^2}{(R+h)}$

$$=gR\left(1-\frac{R}{R+h}\right)=gR\left(\frac{h}{R+h}\right)$$

or $v^2(R+h)=2gRh$

or $Rv^2=2gRh-v^2h=(2gR-v^2)h$

or $h=\frac{Rv^2}{(2gR-v^2)}$

$$=\frac{6.4\times10^6\times(5\times10^3)^2}{(2\times9.8\times6.4\times10^6)-(5\times10^3)^2}$$

$=1.6\times10^6$ m

27. (b): The orbital velocity of satellite is

$$v_0=\sqrt{\frac{GM}{R+h}}$$

$$=\left[\frac{(6.67\times10^{-11})\times(6\times10^{24})}{6.4\times10^6+1.6\times10^6}\right]^{1/2}=7.1\times10^3\ \text{m s}^{-1}$$

28. (a): Since source is moving towards listener. Hence, apparent frequency of sound heard,

$$\upsilon'=\frac{v\times\upsilon}{v-v_s}=\frac{340\times256}{(340-20)}=272\ \text{Hz}$$

29. (b): When the source moves at 90° to the line joining the source and the listener, apparent frequency remains unaffected.

30. (c) : (P) As, horizontal range = 3 × greatest height

$\therefore\ \frac{u^2\sin2\theta}{g}=3\times\frac{u^2\sin^2\theta}{2g}$

or $2\sin\theta\cos\theta=\frac{3}{2}\sin^2\theta$

or $\tan\theta=\frac{4}{3},\ \sin\theta=\frac{4}{5}$ and $\cos\theta=\frac{3}{5}$

$\therefore$ Horizontal range $=\frac{u^2}{g}\times2\sin\theta\cos\theta$

$$=\frac{u^2}{g}\times2\times\frac{4}{5}\times\frac{3}{5}=\frac{24\,u^2}{25\,g}$$

(Q) As, maximum height = 3 × horizontal range

$\therefore\ \frac{u^2\sin^2\theta}{2g}=\frac{3u^2 2\sin\theta\cos\theta}{g}$

or $\tan\theta=12$.

Now, $\sin\theta=\frac{12}{\sqrt{145}}$

and $\cos\theta=\frac{1}{\sqrt{145}}$

$\therefore$ Horizontal range $=\frac{u^2}{g}2\sin\theta\cos\theta$

$$=\frac{u^2}{g}\times2\times\frac{12}{\sqrt{145}}\times\frac{1}{\sqrt{145}}=\frac{24\,u^2}{145\,g}$$

(R) As horizontal range = 2 × greatest height

$\Rightarrow\ \frac{u^2}{g}\times2\sin\theta\cos\theta=2\times\frac{u^2\sin^2\theta}{2g}$

$\therefore\ \tan\theta=2.$

Now $\sin\theta=\frac{2}{\sqrt5}$ and $\cos\theta=\frac{1}{\sqrt5}$

$\therefore$ Horizontal range $=\frac{u^2}{g}\times2\times\frac{2}{\sqrt5}\times\frac{1}{\sqrt5}$

$$=\frac{4\,u^2}{5\,g}$$

(S) As, maximum height = 2 × horizontal range

$\therefore \quad \dfrac{u^2\sin^2\theta}{2g} = 2\times\dfrac{u^2}{g}2\sin\theta\cos\theta$

$\therefore \quad \tan\theta = 8,$

Now $\sin\theta = \dfrac{8}{\sqrt{65}}$ and $\cos\theta = \dfrac{1}{\sqrt{65}}$

$\therefore$ Horizontal range $= \dfrac{u^2}{g}\times 2\times\dfrac{8}{\sqrt{65}}\times\dfrac{1}{\sqrt{65}}$

$= \dfrac{16\,u^2}{65\,g}$

31. (c) : Here, time of ascent + time of descent = 4 s
Therefore, time of ascent = time of descent = 2 s
It means the ball is at the highest position after 2 s.
Taking vertical upward motion of the ball upto the highest position, we have

$a = -9.8 \text{ m s}^{-2}, t = 2 \text{ s}, v = 0$

$v = u + at$

or $0 = u - 9.8\times 2$

or $u = 19.6 \text{ m s}^{-1}$

(P) $s = ut + \frac{1}{2}at^2 = 19.6\times 2 + \frac{1}{2}(-9.8)\times 2^2 = 19.6 \text{ m}$

(Q) When $t = 3$ s, then

$s = 19.6\times 3 + \frac{1}{2}(-9.8)\times 3^2 = 14.7 \text{ m}$

For speed, use $v = u + at$

(R) When $t = 3$ s, then $v = 19.6 + (-9.8)\times 3$
$= -9.8 \text{ m s}^{-1}$

(S) When $t = 4$ s, then $v = 19.6 + (-9.8)\times 4$
$= -19.6 \text{ m s}^{-1}$

32. (b): As velocity

$v = \dfrac{dy}{dt} = \omega\sqrt{A^2 - y^2}$ ... (i)

and acceleration

$a = \dfrac{d^2y}{dt^2} = -\omega^2 y$ ... (ii)

When $y = 0$

velocity, $\dfrac{dy}{dt} = \omega A$ (maximum)

and accelertaion $\dfrac{d^2y}{dt^2} = 0$ (minimum)

when $y = A$, $v = \dfrac{dy}{dt} = 0$ (minimum)

and acceleration $\dfrac{d^2y}{dt^2} = -\omega^2 A$ (maximum)

Hence, acceleration of a particle executing SHM is zero (where velocity is maximum) at mean position and maximum at extreme position where velocity is minimum.

Now, let $y = a\sin(\omega t + \phi)$

Then velocity $\dfrac{dy}{dt} = a\omega\cos(\omega t + \phi)$

$= a\omega\sin\left\{(\omega t + \phi) + \dfrac{\pi}{2}\right\}$

Thus, displacement and velocity of SHM differ by $\dfrac{\pi}{2}$ in phase.

33. (b): For an isothermal process, PV = constant.
Differentiating both sides, $PdV + VdP = 0$.

or $PdV = -VdP$ or $P = -\dfrac{dP}{dV/V} = K_i$ ... (i)

For an adiabatic process, PV^γ = constant
Differentiating both sides,

$\gamma PV^{\gamma-1}dV + V^\gamma dP = 0$ or $\gamma PdV + VdP = 0$

or $P = \dfrac{-dP}{\gamma dV/V} = \dfrac{1}{\gamma}K_a$ ...(ii)

From equation (i) and (ii),

$K_a = \gamma K_i$

As $\gamma > 1$, therefore $K_a > K_i$

34. (a) : With increase in depth the pressure increases from the formula $P \propto h$, therefore, the force perpendicular to the wall of dam increases. Hence, the dam must have greater strength at base than at top. Due to this dams are make thicker at the base than at the top.

35. (c) : From the formula,

Excess pressure, $P = \dfrac{2T}{r}$

Here, T is surface tension, r is the radius of liquid drop. Hence, excess pressure is inversely proportional to radius and hence, the surface area. Therefore, the excess pressure inside a smaller drop is large as compared to the larger drop due to which smaller drop of liquid resists deforming forces better than a larger drop.

36. (2) : Since, volume of bigger drop
= 2 × volume of smaller drop.

$\therefore \quad \dfrac{4\pi}{3}R^3 = 2\left(\dfrac{4\pi}{3}r^3\right) \Rightarrow R = 2^{1/3}r$

As terminal velocity, $v \propto r^2$,

$\therefore \quad \dfrac{V}{v} = \dfrac{R^2}{r^2} = \dfrac{2^{2/3}r^2}{r^2} = 2^{2/3}$

Clearly, $V = 2^{2/3}\,v = 2^{2/3}\,(2^{1/3} \text{ m s}^{-1}) = 2 \text{ m s}^{-1}$

37. (0) : Here, $\dfrac{\Delta L}{L} = 2\times 10^{-3}, \sigma = 0.5$

As $\sigma = -\dfrac{\Delta D / D}{\Delta L / L}$,

$\therefore \quad \dfrac{\Delta D}{D} = -\sigma\left(\dfrac{\Delta L}{L}\right)$

$= -0.5\ (2 \times 10^{-3}) = -1 \times 10^{-3}$

As $V = \left(\dfrac{\pi D^2}{4} L\right)$,

$$\therefore \quad \frac{\Delta V}{V} = \frac{\Delta\left(\dfrac{\pi D^2}{4} L\right)}{\dfrac{\pi D^2}{4} L} = \frac{D^2 \times \Delta L + L \times 2D \times \Delta D}{D^2 L}$$

or $\dfrac{\Delta V}{V} \times 100 = \dfrac{\Delta L}{L} \times 100 + \left(2\dfrac{\Delta D}{D}\right) \times 100$

$= (2 \times 10^{-3} + 2\ (-1 \times 10^{-3})\% = 0\ \%$

38. (1) : Here, $K = \dfrac{1}{2}E$,

$\therefore \quad \dfrac{1}{2}m\omega^2 y^2 = \dfrac{1}{2}\left(\dfrac{1}{2}m\omega^2 A^2\right)$

or $y = \dfrac{1}{\sqrt{2}} A$

Also, $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{8} = \dfrac{\pi}{4}$ rad s^{-1} ...(i)

As equation of SHM,

$y = A\cos(\omega t)$,

or $\dfrac{1}{\sqrt{2}} A = A\cos(\omega t)$

or $\cos \omega t = \dfrac{1}{\sqrt{2}} = \cos\left(\dfrac{\pi}{4}\right)$

$\Rightarrow \quad \omega t = \dfrac{\pi}{4}$

$\therefore$ From equation (i)

$\left(\dfrac{\pi}{4}\right)t = \left(\dfrac{\pi}{4}\right)$ or $t = 1$ s

39. (8) : As $P = Fv = \left(\dfrac{dp}{dt}\right)v \quad [\because$ Momentum, $p = mv]$

$= \left[\dfrac{d}{dt}(mv)\right]v = \left(v\dfrac{dm}{dt}\right)v = v^2\dfrac{dm}{dt}$ ... (i)

But $\dfrac{dm}{dt}$ = mass of water flowing per second

= Volume of water flowing per second × density of water

$\dfrac{dm}{dt} = V\rho$ ... (ii)

Further, if a is the area of cross-section of the pipe,

$V = av$ or $v = \dfrac{V}{a}$ ... (iii)

From eqns. (i), (ii) and (iii),

$P = \left(\dfrac{V}{a}\right)^2 V\rho = \dfrac{V^3\rho}{a^2}$, *i.e.*, $P \propto V^3$

Thus, for V to become $2V$, P should be $(2^3)P$, *i.e.*, power of the motor should be increased 8 times.

40. (4) : Total KE of the hoop,

$K = K_t$ (KE of translation) + K_r (KE of rotation)

$= \dfrac{1}{2}mv^2 + \dfrac{1}{2}I\omega^2$

$= \dfrac{1}{2}mv^2 + \dfrac{1}{2}\left(\dfrac{1}{2}mr^2\right)\omega^2 = mv^2$

(as I = *MI* of the hoop of radius r about an axis perpendicular to its plane $= \dfrac{1}{2}mr^2$)

or $K = (100)(0.2)^2$ J = 4 J

$(\because v = 20$ cm s^{-1} = 0.2 m s$^{-1})$

Work required to be done to stop the hoop = K = 4 J

Note : The radius of the hoop has no role to play in this case.

DO YOU KNOW?

- Because of differences in gravity, a 200 pound person would only weigh 76 pounds on Mars.
- Electric eels can stun both predators and prey with electric shocks of around 500 volts.
- Energy from food is usually measured in joules or calories.
- Light from the Earth takes just 1.255 seconds to reach the Moon.
- Sound travels at a speed of around 767 miles per hour (1,230 kilometres per hour).
- When traveling at 80 kilometres per hour (50 miles per hour), cars use around half of their fuel just to overcome wind resistance.
- Water can work against gravity, moving up narrow tubes in a process called capillary action.
- A magnifying glass uses the properties of a convex shaped lens to magnify an image, making it easier to see.
- A scientist who studies physics is known as a physicist.
- Uranus is the only planet in our solar system that rolls on its side like a barrel, while Venus is the only planet that spins in the opposite direction to Earth.
- The fastest land animal in the world is the Cheetah, clocking a max speed of around 113 km per hour (70 mph).
- 1921 Nobel Prize in Physics was won by Albert Einstein for his work in the field of theoretical physics.

SAMPLE PAPER

CBSE CLASS XII

2014

BOARD EXAMINATION

1. A hollow metal sphere of radius 10 cm is charged such that the potential on its surface is 5 V. What is the potential at the centre of the sphere? **[1]**

2. A toaster produces more heat than a light bulb when they are connected in parallel to a 220 V mains. Which has the greater resistance? **[1]**

3. Why do we prefer steel or alnico for making permanent magnets? **[1]**

4. In India, domestic power supply is at 220 V-50 Hz, while in USA, it is 110 V-50 Hz. Give one advantage and one disadvantage of 220 V supply over 110 V supply. **[1]**

5. How will you represent a resistance of $3700\ \Omega \pm 10\%$ by colour code? **[1]**

6. What type of wavefront will emerge from (i) a point source and (ii) distant light source? **[1]**

7. What is the basic difference between an analog communication system and digital communication system? **[1]**

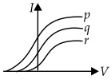
8. What should be the length of the dipole antenna for a carrier wave of frequency 3×10^8 Hz? **[1]**

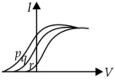
9. The output of a two input AND gate is connected to both the inputs of a NAND gate. Draw the logic circuit of this combination of gates and write its truth table. **[2]**

10. Light of wavelength 3500 Å is incident on two metals *A* and *B*. Which metal yields photoelectrons if their work functions are 4.2 eV and 1.9 eV respectively?

OR

Photoelectric effect experiments are performed using three different metal plates *p*, *q* and *r* having work function, $\phi_p = 2.0$ eV, $\phi_q = 2.5$ eV and $\phi_r = 3.0$ eV respectively. A light beam containing wavelengths of 550 nm, 450 nm and 350 nm with equal intensities illuminates each of the plates. The correct *I*-*V* graph for the experiment is
(Take $hc = 1240$ eV nm)

(a)


(b)


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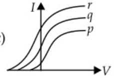

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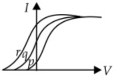
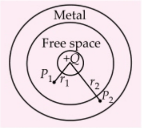


[2]

11. Electrons are continuously in motion within a conductor but there is no current in it unless some source of potential is applied across its ends. Give reason. **[2]**

12. A small metal sphere carrying charge $+Q$ is located at the centre of a spherical cavity in a large uncharged metal sphere as shown in figure. Use Gauss's theorem to find electric field at the points P_1 and P_2.



[2]

13. Two coherent sources, whose intensity ratio is 81 : 1, produce interference fringes. Calculate the ratio of intensities of maxima and minima in the fringe system. **[2]**

14. Write the relation for the force $\vec{F}$ acting on a charge particle having charge q moving with a velocity $\vec{v}$ through a magnetic field $\vec{B}$ in vector notation. Using this relation, deduce the conditions under which this force will be (i) maximum (ii) minimum. **[2]**

15. Two point charges, $q_1 = 10 \times 10^{-8}$ C and $q_2 = -2 \times 10^{-8}$ C are separated by a distance of 60 cm in air.

(i) Find at what distance from the 1st charge q_1, would the electric potential be zero.

(ii) Also calculate the electrostatic potential energy of the system. **[2]**

16. A 15 μF capacitor is connected to 220 V, 50 Hz source. Find the peak current. **[2]**

17. Draw a graph showing the variation of stopping potential with frequency of incident radiation. What does the slope of the line with frequency axis indicate? **[2]**

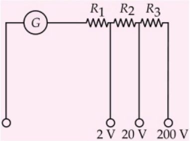
18. A straight horizontal conducting rod of length 0.45 m and mass 60 g is suspended by two vertical wires at its ends. A current of 5 A is set up in the rod through the wires. (a) What magnetic field should be set up normal to the conductor in order that the tension in the wires is zero? (b) What will be the total tension in the wires if the direction of current is reversed, keeping the magnetic field same as before. (Ignore the mass of the wire, Take $g = 9.8$ m s^{-2}). **[2]**

19. The plates of a parallel plate capacitor have an area of 90 cm^2 each and are separated by 2.5 mm. The capacitor is charged by connecting it to a 400 V supply.

(a) How much electrostatic energy is stored by the capacitor?

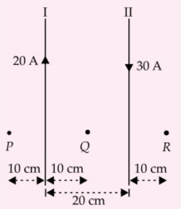
(b) View this energy as stored in the electrostatic field between the plates, and obtain the energy per unit volume (u). Hence arrive at a relation between u and the magnitude of electric field E between the plates. **[3]**

20. A multirange voltmeter can be constructed by using a galvanometer circuit as shown in figure. We want to construct a voltmeter that can measure 2 V, 20 V and 200 V using a galvanometer of resistance 10 Ω and that produces maximum deflection for current of 1 mA. Find R_1, R_2 and R_3 that have to be used.



OR

In the figure two long parallel current carrying wires I and II are shown. Find the magnitudes and directions of the magnetic field induction at the points P, Q and R in the plane of paper.



[3]

21. Due to economic reasons, only the upper side band of an AM wave is transmitted, but at the receiving station, there is a facility for generating the carrier. Show that if device is available which can multiply two signals, then it is possible to recover the modulating signal at the receiver station. **[3]**

22. A double convex lens made of glass of refractive index 1.6 has its both surfaces of equal radii of curvature of 30 cm each. An object of 5 cm height is placed at a distance of 12.5 cm from the lens. Find the position, nature and size of the image. **[3]**

23. Calculate the longest and shortest wavelength in the Balmer series of hydrogen atom. Given Rydberg constant = 1.097×10^7 m^{-1}. **[3]**

24. Ram was a student of science and was suffering from disease. He was under treatment of a registered medical practitioner. The doctor sent Ram repeatedly for X-ray examination. Ram was hesitant for the same. He told the doctor that they had been taught that the repeated exposure to X-rays would be harmful. The doctor told him not to worry as he knew things better.

Read the above passage and answer the following questions.

(i) For what purpose X-ray examination of a patient is required by a doctor?

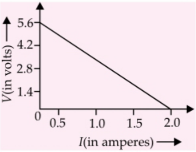
(ii) Is the doctor right to ask Ram for repeated X-ray examination?

(iii) What do you learn from this study? **[3]**

25. A myopic adult has a far point at 0.1 m. His power of accommodation is 4 diopters. (i) What power lenses are required to see distant object? (ii) What is his near point without glasses? (iii) What is his near point with glasses? Take the image distance from the lens of the eye to the retina to be 2 cm. **[3]**

26. Suppose India has a target of producing by 2020 A.D., 2×10^5 MW of electric power, ten percent of which is to be obtained from nuclear power plants. Suppose we are given that on an average, the efficiency of utilization (*i.e.*, conversion to electrical energy) of thermal energy produced in a reactor is 25%. How much amount of fissionable uranium will our country need per year? Take the heat energy per fission of U^{235} to be about 200 MeV. **[3]**

27. 4 cells of identical emf ε, internal resistance r are connected in series to a variable resistor. The following graph shows the variation of terminal voltage of the combination with the current output :



(i) What is the emf of each cell used?

(ii) Calculate the internal resistance of each cell.

(iii) For what current from the cells, does maximum power dissipation occur in the circuit? **[3]**

28. Give the principle, construction, theory and working of an ac generator.

OR

Derive an expression for the average power over a complete cycle of ac in a series *LCR* circuit connected to an ac source in which the phase difference between the voltage and current in the circuit is ϕ. **[5]**

29. Describe diffraction of light due to a single slit and obtain an expression for width of central maximum.

OR

Describe Young's double slit experiment for interference and obtain expression for fringe width. **[5]**

30. Draw the energy band diagrams of *p*-type and *n*-type semiconductors. Explain with a circuit diagram the working of full-wave rectifier.

OR

Draw a labelled circuit diagram of a common emitter amplifier using a *p-n-p* transistor. Explain how the input and output voltage are out of phase by 180° for a common-emitter transistor amplifier. Define the term voltage gain and write an expression for it. **[5]**

SOLUTIONS

1. As $E = -\dfrac{dV}{dr}$ and for a hollow metal sphere, $E = 0$ at the centre,

$\therefore \; \dfrac{dV}{dr} = 0$ or $V = \text{constant} = 5\text{ V}$

2. Since both the light bulb and the toaster are connected in parallel, the potiental difference across both is the same, *i.e.*, 220 V.
Further, as the toaster produces more heat than the bulb,

$$\frac{V^2}{R} > \frac{V^2}{R'} \text{ or } \frac{1}{R} > \frac{1}{R'}$$

or $R' > R$

Thus, the resistance of the bulb (R') is more than that of the toaster (R).

3. A permanent magnet should retain its magnetism for a long time. As such, it should be of a material of high coercivity, which is so in case of steel or alnico.

4. Advantage : The power loss ($I_{rms}^2 R$) is less at 220 V supply than that at 110 V (due to smaller value of I_{rms} in the former case).
Disadvantage : The peak value of 220 V is $220\sqrt{2} = 311\, V$, whereas that of 110 V is 155.6 V. Obviously, 220 V supply is more dangerous than 110 V supply.

5. The value of carbon resistance = 3700 Ω ± 10% = $37 \times 10^2 \pm 10\%$
The colour assigned to number 3, 7 and 2 are orange, violet and red respectively. For ± 10% accuracy, the colour is silver. Thus, the bands of colour on carbon resistance in sequence are orange, violet, red and silver.

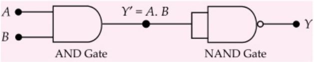
6. (i) From a point source, the wavefront is diverging spherical wavefront.
(ii) From a distant light source, the wavefront is plane wavefront.

7. An analog communication system makes use of analog signals, which vary continuously with time. A digital communication system makes use of a digital signal, which has only two values of voltage either high or low.

8. As $\lambda = \dfrac{c}{\upsilon} = \dfrac{3 \times 10^8 \text{ m s}^{-1}}{3 \times 10^8 \text{ Hz}} = 1\text{ m}$,

Length of the dipole antenna $= \frac{\lambda}{2} = 0.5$ m

9. The logic circuit for the given combination of gates is shown in the given figure.



Truth Table

A	B	$Y' = A \cdot B$	$Y = \overline{Y' \cdot Y'}$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

10. Here, $\lambda = 3500$ Å $= 3500 \times 10^{-10}$ m

Let E be the energy of the incident photon.

$$E = h\upsilon = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34})(3\times10^{8})}{(3500\times10^{-10})(1.6\times10^{-19})}\text{ eV}$$

or $\quad E = 3.55$ eV

For metal A, $E < 4.2$ eV and for metal B, $E > 1.9$ eV. Thus metal B yields photoelectrons with the light of given wavelength.

OR

For photoelectric emission to take place, the wavelength of incident light should be less than the threshold wavelength for that metal. If λ_m is the threshold wavelength, then work function of metal

$$\phi = \frac{hc}{\lambda_m} \quad\text{or}\quad \lambda_m = \frac{hc}{\phi}$$

For metal p, $\lambda_{m_p} = \frac{1240 \text{ eV nm}}{2.0 \text{ eV}} = 620$ nm

For metal q, $\lambda_{m_q} = \frac{1240 \text{ eV nm}}{2.5 \text{ eV}} = 496$ nm

For metal r, $\lambda_{m_r} = \frac{1240 \text{ eV nm}}{3.0 \text{ eV}} = 413.3$ nm

Wavelengths in the incident beam are 550 nm, 450 nm and 350 nm. Thus the photoelectric emission is possible by light of wavelength 350 nm (being shorter than λ_m) from metals p, q and r. For light of wavelength 450 nm (< 620 nm and 496 nm) photoelectric emission is possible from metals p and q. For light of wavelength 550 nm (< 620 nm), the photoelectric emission is possible from metal p.

Current (I) $\propto$ Intensity

where Intensity $= Nh\upsilon$ of photoelectrons

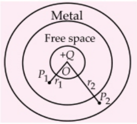
$\therefore$ I is maximum for p metal, less for q metal and least for r metal. Therefore the graph (a) is correct.

11. In the absence of any external voltage source, the motion of electrons in a conductor is random and electrons collide continuously with the positive ions of metal. This causes a random change in direction. The average velocity of random motion of electrons in any direction is zero, hence current in conductor without external source is zero.

12. Consider a spherical surface of radius r_2, such that $OP_2 = r_2$. Charge induced on inner surface of metallic sphere $= -Q$

Charge induced on outer surface of metallic sphere $= +Q$

$\therefore$ Net charge enclosed by surface $S_2 = Q - Q = 0$



$\therefore$ By Guass's theorem, total electric flux through surface S_2

$$= \frac{1}{\varepsilon_0} \times \text{net charge enclosed.}$$

$$E_2 . 4\pi r^2 = \frac{1}{\varepsilon_0} \times 0 \quad \therefore \quad E_2 = 0$$

i.e., electric field at P_2 is zero.

Similarly, electric field at P_1 is also zero.

13. Here, $\frac{I_1}{I_2} = \frac{A_1^2}{A_2^2} = \frac{81}{1} \quad \therefore \quad \frac{A_1}{A_2} = \frac{9}{1}$

$$\text{Now, } \frac{I_{max}}{I_{min}} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2} = \frac{A_2^2\left(\frac{A_1}{A_2} + 1\right)^2}{A_2^2\left(\frac{A_1}{A_2} - 1\right)^2}$$

$$= \frac{(9+1)^2}{(9-1)^2} = \frac{100}{64} = \frac{25}{16}$$

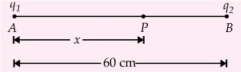
14. The force acting on a charged particle having charge q moving with velocity $\vec{v}$ through a magnetic field $\vec{B}$ is given by

$\vec{F} = q(\vec{v} \times \vec{B})$ or $F = qvB\sin\theta$

where θ is the angle between $\vec{v}$ and $\vec{B}$.

(i) Force will be maximum when $\sin\theta = 1$ or $\theta = 90°$ *i.e.*, charge is moving perpendicular to the magnetic field.

(ii) Force will be minimum when $\sin\theta = 0$, or $\theta = 0°$ or $180°$ *i.e.*, charge is moving either parallel or antiparallel to the magnetic field.

15. 

(i) Here, $q_1 = 10 \times 10^{-8}$ C, $q_2 = -2 \times 10^{-8}$ C
and $AB = 60$ cm $= 0.6$ m
Let x be the distance of point P from charge q_1 at which electric potential is zero.
Potential at P is

$$V_P = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{AP} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{PB} = 0$$

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\frac{(10\times10^{-8})}{x} + \frac{1}{4\pi\varepsilon_0}\frac{(-2\times10^{-8})}{(0.6-x)} = 0$$

$$\frac{10}{x} = \frac{2}{0.6-x}, \; 6 - 10x = 2x \quad \text{or} \quad 12x = 6$$

$x = 0.5$ m $= 50$ cm

(ii) Electrostatic potential energy of the system

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$

$$U = 9\times10^9 \times \frac{10\times10^{-8}\times(-2\times10^{-8})}{0.6}$$

$$U = -\frac{18\times10^{-6}}{0.6}, U = -30\times10^{-6} = -3\times10^{-5}\text{ J}$$

16. Here, $C = 15\ \mu$F $= 15 \times 10^{-6}$ F,

$V_v = 220$ V, $\upsilon = 50$ Hz,

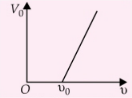
$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi \upsilon C}$$

$$= \frac{1}{2\times\frac{22}{7}\times50\times15.0\times10^{-6}} = 212.1\ \Omega$$

$$I_v = \frac{V_v}{X_C} = \frac{220}{212.1} = 1.037\text{ A}$$

$$I_0 = \sqrt{2}I_v = 1.414\times1.037\text{ A} = 1.47\text{ A}$$

17. The V_0-υ graph is a straight line as shown in the figure.



$eV_0 = h\upsilon - W_0$

$$V_0 = \frac{h}{e}\upsilon - W_0$$

Comparing the above relation with the equation of straight line, $y = mx + c$

$\therefore$ The slope of V_0 - υ graph is $\frac{h}{e}$.

18. Here, $l = 0.45$ m , $m = 60$ g $= 60 \times 10^{-3}$ kg, $I = 5$ A.
(a) Tension in the wire is zero if force on the wire carrying current due to magnetic field is equal and opposite to the weight of wire

i.e. $BIl = mg$ or $B = \frac{mg}{Il}$

$$= (60\times10^{-3})\frac{9.8}{5.0\times.45} = 0.26\text{ T}$$

The force due to magnetic field will be upwards if the direction of field is horizontal and normal to the conductor.
(b) When direction of current is reversed, BIl and mg will act vertically downwards, the effective tension in the wires,
$T = BIl + mg = 0.26 \times 5 \times 0.45 + (60 \times 10^{-3}) \times 9.8$
$= 1.173$ N

19. Here, $A = 90$ cm^2 $= 90 \times 10^{-4}$ m^2 $= 9 \times 10^{-3}$ m^2, $d = 2.5$ mm $= 2.5 \times 10^{-3}$ m, $V = 400$ V

(a) $\quad U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{\epsilon_0 A}{d}V^2;$

$$u = \frac{8.85\times10^{-12}\times9\times10^{-3}(400)^2}{2\times2.5\times10^{-3}} = 2.55\times10^{-6}\text{ J}$$

(b) Volume of capacitor,
$V = A \times d = 90 \times 10^{-4} \times 25 \times 10^{-3}$ m^3 $= 2.25 \times 10^{-4}$m^3
Energy per unit volume,

$$u = \frac{2.55\times10^{-6}}{2.25\times10^{-4}} = 0.113\text{ J m}^{-3}$$

As $\quad u = \frac{U}{V} = \frac{\frac{1}{2}CV^2}{Ad} = \frac{\frac{\epsilon_0 A}{2d}V^2}{Ad} = \frac{1}{2}\epsilon_0\left(\frac{V}{d}\right)^2$

But $\frac{V}{d} = E$, electric intensity field intensity)

$\therefore \quad u = \frac{1}{2} = \epsilon_0 E^2$

20. Here, $G = 10\ \Omega$, $I_g = 1$ mA $= 10^{-3}$A
Case (i), $V = 2$ V

$$R_1 = \frac{V}{I_g} - G = \frac{2}{10^{-3}} - 10 = 1990\ \Omega \approx 2\text{ k}\ \Omega$$

Case (ii), $V = 20$ V

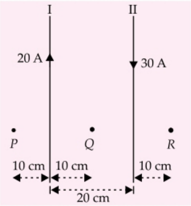
$$(R_1 + R_2) = \frac{20}{10^{-3}} - 10 = 20{,}000 - 10 \approx 20\text{ k}\Omega$$

$\therefore \quad R_2 = 20$ kΩ $- 2$ kΩ $= 18$ kΩ
Case (iii), $V = 200$ V

$$\therefore \quad R_1 + R_2 + R_3 = \frac{200}{10^{-3}} - 10 \approx 200\text{ k}\ \Omega$$

$\therefore \quad R_3 = 200$ kΩ $- 20$ kΩ ≈ 180 kΩ

OR



Resultant magnetic field induction at P is

$$B = B_1 - B_2 = \frac{\mu_0}{4\pi}\frac{2I_1}{r_1} - \frac{\mu_0}{4\pi}\frac{2I_2}{r_2}$$

$$= \frac{\mu_0}{4\pi} \times 2\left[\frac{I_1}{r_1} - \frac{I_2}{r_2}\right]$$

Here, $I_1 = 20$ A, $r_1 = 0.1$ m;

$I_2 = 30$ A; $r_2 = 0.30$ m

$$B = 10^{-7} \times 2\left[\frac{20}{0.1} - \frac{30}{0.3}\right] = 2 \times 10^{-5}\ \text{T}$$

It will be acting perpendicular to the plane of paper upwards.

Resultant magnetic field induction at Q is

$$B = \frac{\mu_0}{4\pi} \times 2\left[\frac{20}{0.1} + \frac{30}{0.1}\right]$$

$$= 10^{-7} \times 2 \times 500 = 10^{-4}\ \text{T}$$

It will be acting perpendicular to the plane of the paper downwards.

Resultant magnetic field induction at R is

$$B = \frac{\mu_0}{4\pi} \times 2\left[\frac{30}{0.1} - \frac{20}{0.3}\right] = 4.7 \times 10^{-5}\ \text{T}$$

It will be acting perpendicular to the plane of the paper upwards.

21. Let ω_c and ω_m be the angular frequency of carrier waves and message signal wave respectively.
For simplicity, let the signal received at the receiving station be

$$A = A_1\cos(\omega_c + \omega_m)t$$

Let the instantaneous voltage of carrier waves (c) given by

$$C = A_c\cos\omega_c t$$

be available at the receiving station,

Now, $A \times C = A_1A_c\cos(\omega_c + \omega_m)t\cos\omega_c t$

$$= \frac{A_1A_c}{2}[\cos\{(\omega_c + \omega_m)t + \omega_c t\} + \cos\{(\omega_c + \omega_m)t - \omega_c t\}]$$

$$= \frac{A_1A_c}{2}[\cos(2\omega_c + \omega_m)t + \cos\omega_m t].$$

At the receiving station, when the signal is passed through low-by pass filter, it will pass the high frequency signals $(2\omega_c + \omega_m)$ but obstruct the low frequency signal ω_m. Therefore, we can record the modulating signal $= \frac{A_1A_c}{2}\cos\omega_m t$, which is a signal of angular frequency ω_m.

22. According to lens maker's formula

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

Here, $\mu = 1.6$, $R_1 = 30$ cm, $R_2 = -30$ cm,

$$\therefore \quad \frac{1}{f} = (1.6 - 1)\left[\frac{1}{30} + \frac{1}{30}\right]$$

$$\frac{1}{f} = (1.6 - 1)\left[\frac{2}{30}\right] = \frac{1.2}{30}$$

$$f = 25\ \text{cm}$$

According to thin lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{25} - \frac{1}{12.5}$$

$$v = -25\ \text{cm}$$

Negative sign shows that the image is virtual.

As $\quad m = \dfrac{h_I}{h_O} = \dfrac{v}{u}$

$$\therefore \quad h_I = \frac{v}{u} \times h_O = \frac{(-25)}{(-12.5)} \times 5 = 10\ \text{cm}$$

The image is erect, virtual and magnified.

23. The wavelength (λ) of different spectral lines of Balmer series is given by

$$\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{n^2}\right] \qquad \text{where } n = 3, 4, 5, 6, ...$$

For longest wavelength, $n = 3$

$$\therefore \quad \frac{1}{\lambda} = 1.097 \times 10^7\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = 1.097 \times 10^7 \times \frac{5}{36}$$

$$\lambda = \frac{36}{5 \times 1.097 \times 10^7}\ \text{m}$$

$$\lambda = \frac{36 \times 10^{10}}{5 \times 1.097 \times 10^7}\ \text{Å}$$

$$\lambda = 6563\ \text{Å}$$

For shortest wavelength, $n = \infty$

$$\therefore \quad \frac{1}{\lambda} = 1.097 \times 10^7\left[\frac{1}{2^2} - \frac{1}{\infty^2}\right] = \frac{1.097 \times 10^7}{4}$$

$$\lambda = \frac{4}{1.097 \times 10^7}\ \text{m} = \frac{4 \times 10^{10}\ \text{Å}}{1.097 \times 10^7} = 3646\ \text{Å}$$

24. (i) X-ray examination of a patient is required to diagnose the location of a defect or disease especially in bones, as X-rays can pass through blood and flesh but not through bones.
(ii) No, the doctor is not right as repeated X-ray examinations of a human body may destroy small and sensitive tissues of the body which may cause further problems.
(iii) The doctor is not expected to misguide a patient for extraneous considerations. For a patient, the doctor is next to God. He should always provide a fair treatment.

25. (i) Distance of far point, $u = -0.1$ m, distance of image, $v = 2$ cm $= 0.02$ m

$\therefore$ Power of myopic eye,

$$P_f = \frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{0.02} + \frac{1}{0.1} = 50 + 10 = 60 \text{ D}$$

With the corrective lens, far point shift to ∞.

$\therefore$ Power required, $P'_f = \dfrac{1}{0.02} - \dfrac{1}{\infty} = 50$ D

Required power of glasses, $P_g = P'_f - P_f = 50 - 60$
$= -10$ D.

(ii) Power of accommodation $= 4$ D

If P_n is power of normal eye for near vision, then

$4 = P_n - P_f = P_n - 60$ or $P_n = 64$ D

If x_n is near point without glasses, then

$$\frac{1}{x_n} + \frac{1}{0.02} = 64 \text{ or } \frac{1}{x_n} = 14,\ x_n = \frac{1}{14} \text{ m} = 0.07 \text{ m}$$

(iii) With glasses, $P'_n = P'_f + 4 = 50 + 4 = 54$

$$\therefore \quad \frac{1}{x'_n} + \frac{1}{0.02} = 54$$

$$\frac{1}{x'_n} = 54 - 50 = 4$$

$$x'_n = \frac{1}{4} \text{ m} = 0.25 \text{ m}$$

26. Total target power $= 2 \times 10^5$ MW

Total nuclear power $= 10\%$ of 2×10^5 MW
$= 2 \times 10^4$ MW.

Energy produced per fission $= 200$ MeV

Efficiency of power plant $= 25\%$

$\therefore$ Energy converted into electrical energy per fission $= \dfrac{25}{100} \times 200 = 50 \text{ MeV} = 50 \times 1.6 \times 10^{-13}$ J s

Total electrical energy to be produced

$= 2 \times 10^4$ MW $= 2 \times 10^4 \times 10^6$ W $= 2 \times 10^{10}$ J s^{-1}

$= 2 \times 10^{10} \times 60 \times 60 \times 24 \times 365$ joule per year

Number of fission in one year

$$= \frac{2 \times 10^{10} \times 60 \times 60 \times 24 \times 365}{50 \times 1.6 \times 10^{-13}}$$

$$= 2 \times \frac{36 \times 24 \times 365}{8} \times 10^{24}$$

Mass of 6.023×10^{23} atoms of $U^{235} = 235$ g
$= 235 \times 10^{-3}$ kg

Mass of $\dfrac{2 \times 36 \times 24 \times 365}{8} \times 10^{24}$ atoms

$$= \frac{235 \times 10^{-3}}{0.023 \times 10^{23}} \times \frac{2 \times 36 \times 24 \times 365 \times 20^{24}}{8}$$

$= 3.08 \times 10^4$ kg

Hence mass of uranium needed per year
$= 3.08 \times 10^4$ kg

27. Emf of each cell $= \varepsilon$

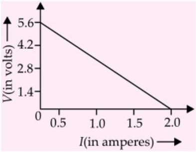
Internal resistance of each cell $= r$

Number of cells $(n) = 4$

Total emf $= n\varepsilon = 4\varepsilon$

Total internal resistance $= nr = 4r$

Terminal voltage, $(V) = 4\varepsilon - I(4r)$ ...(i)



(i) From the graph,
when $I = 0$, $V = 5.6$ V

$5.6 = 4\varepsilon - 0$ (using (i))

$$\varepsilon = \frac{5.6}{4} \text{ V} = 1.4 \text{ V}$$

Emf of each cell, $\varepsilon = 1.4$ V

(ii) From graph, when $V = 0$, $I = 2.0$ A

$0 = 4\varepsilon - 8r$ (using (i))

$$r = \frac{4\varepsilon}{8} = \frac{4 \times 1.4}{8} = 0.7\ \Omega$$

Internal resistance of each cell, $r = 0.7\ \Omega$

(iii) Power output is maximum when external resistance = effective internal resistance

$\therefore$ For maximum power,
External resistance, $R = 4r = 4 \times 0.7 = 2.8\ \Omega$

$$\text{Current in circuit, } I = \frac{4\varepsilon}{R + 4r} = \frac{4 \times 1.4}{2.8 + 2.8} = 1 \text{ A}$$

28. Refer point 4.8(2) page no. 252 (MTG Excel in Physics).

OR

Refer point 4.6(9) page no. 248 (MTG Excel in Physics).

29. Refer point 6.14 page no. 409 (MTG Excel in Physics).

OR

Refer point 6.13 page no. 406 (MTG Excel in Physics).

30 Refer point 9.2 (2) page no. 530 and point 9.3 (6(ii)) page no. 534 (MTG Excel in Physics).

OR

Refer point 9 page no. 540 (MTG Excel in Physics).

EXAMINER'S MIND NCERT CLASS XII

The questions given in this column have been prepared strictly on the basis of NCERT Physics for Class XII. This year JEE (Main & Advanced)/AIPMT/AIIMS/other PMTs have drawn their papers heavily from NCERT books.

SECTION-1

Only One Option Correct Type

This section contains 14 multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

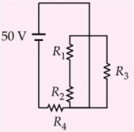
1. Two copper balls, each weighting 10 g, are kept in air 10 cm apart. If one electron from every 10^6 atoms is transferred from one ball to the other, the coulomb force between them is (atomic weight of copper is 63.5)

(a) 2.0×10^{10} N (b) 2.0×10^4 N
(c) 2.0×10^8 N (d) 2.0×10^6 N

2. A torque of 10^{-5} N m is required to hold a magnet at 90° with the horizontal component of the earth's magnetic field. The torque required to hold it at 30° will be

(a) 5×10^{-6} N m (b) $\frac{1}{2} \times 10^{-5}$ N m
(c) $5\sqrt{3} \times 10^{-6}$ N m (d) 2×10^{-6} N m

3. The potential difference in volt across the resistance R_3 in the circuit shown in figure, is (Given $R_1 = 15\ \Omega$, $R_2 = 15\ \Omega$, $R_3 = 30\ \Omega$, $R_4 = 35\ \Omega$)



(a) 5 (b) 7.5
(c) 15 (d) 12.5

4. A source emits sound of frequency 600 Hz inside water. The frequency heard in air will be equal to (velocity of sound in water = 1500 m s^{-1} velocity of sound in air = 300 m s^{-1})

(a) 3000 Hz (b) 120 Hz
(c) 600 Hz (d) 6000 Hz.

5. An alpha particle of energy 5 MeV is scattered through 180° by a fixed uranium nucleus. The distance of closest approach is of the order of

(a) 10Å (b) 10^{-10} cm
(c) 10^{-12} cm (d) 10^{-15} cm

6. An ac voltage source of variable angular frequency ω and fixed amplitude V_0 is connected in series with a capacitance C and an electric bulb of resistance R (inductance zero). When ω is increased

(a) the bulb glows dimmer
(b) the bulb glows brighter
(c) total impedance of the circuit is unchanged
(d) total impedance of the circuit increases

7. An electron of mass m and charge q is travelling with a speed v along a circular path of radius r at right angles to a uniform magnetic field B. If speed of the electron is doubled and the magnetic field is halved, then resulting path would have a radius of

(a) $\frac{r}{4}$ (b) $\frac{r}{2}$ (c) $2r$ (d) $4r$

8. In a free space electron is placed in the path of a plane electromagnetic wave, it will start moving along

(a) centre of earth (b) equator of earth
(c) magnetic field (d) electric field

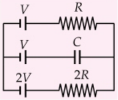
9. The radii of curvature of the two surfaces of a lens are 20 cm and 30 cm and the refractive index of the material of the lens is 1.5. If the lens is concavo-convex, then the focal length of the lens is

(a) 24 cm (b) 10 cm
(c) 15 cm (d) 120 cm

10. A TV tower has a height 150 m. What is the total population covered by the TV tower, if the population density around the TV tower is 10^3 km^{-2}? Radius of the earth is 6.4×10^6 m.

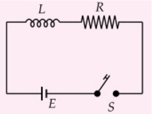
(a) 60.29 lakh (b) 40.192 lakh
(c) 100 lakh (d) 20.22 lakh

11. In the given circuit with steady current, the potential drop across the capacitor must be



(a) V
(b) $\frac{V}{2}$
(c) $\frac{V}{3}$ (d) $\frac{2V}{3}$

12. In the circuit shown below, switch S is closed at a time $t = 0$. The charge which passes through the battery in one time constant is



(a) $\frac{EL}{eR^2}$
(b) $\frac{eL}{ER}$
(c) $\frac{eR^2E}{L}$ (d) $E\left(\frac{L}{R}\right)$

13. A sinusoidal voltage of peak value 300 V and an angular frequency ω = 400 rad s^{-1} is applied to series L-C-R circuit, in which $R = 3\ \Omega$, L = 20 mH and C = 625 μF. The peak current in the circuit is
(a) $30\sqrt{2}$ A (b) 60 A
(c) 100 A (d) $60\sqrt{2}$ A

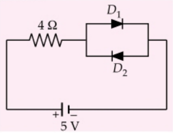
14. The wavelength of the first spectral line in the Balmer series of hydrogen atom is 6561 Å. The wavelength of the second spectral line in the Balmer series of singly-ionized helium atom is
(a) 1215 Å (b) 1640 Å
(c) 2430 Å (d) 4687 Å

SECTION - 2

One or More Options Correct Type

This section contains 9 multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which ONE or MORE are correct.

15. In the network shown, which of the following statements is true?



(a) the potential difference across D_2 is 5 V
(b) current through resistor equals 1.25 A
(c) current through diode D_1 is 1.25 A
(d) current through diode D_2 is 1.25 A

16. A microammeter has a resistance of 100 Ω and a full scale range of 50 μA. It can be used as a voltmeter or as a higher range ammeter provided a resistance is added to it. Pick the correct range and resistance combination(s)
(a) 50 V range with 10 kΩ resistance in series
(b) 10 V range with 200 kΩ resistance in series
(c) 5 mA range with 1 Ω resistance in parallel
(d) 10 mA range with 1 Ω resistance in parallel.

17. A ray of light travelling in a transparent medium falls on a surface separating the medium from air, at an angle of incidence of 45°. The ray undergo total internal reflection. If n is the refractive index of the medium with respect to air, select the possible values of n from the following
(a) 1.3 (b) 1.4
(c) 1.5 (d) 1.6

18. The SI unit of inductance, henry can be written as
(a) weber/ampere
(b) volt-second/ampere
(c) joule/(ampere)^2
(d) ohm-second

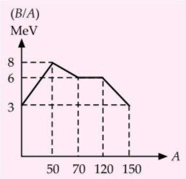
19. When photons of energy 4.25 eV strike the surface of a metal, the ejected photoelectrons have a maximum kinetic energy E_A eV and de-Broglie wavelength λ_A. The maximum kinetic energy of photoelectrons liberated from another metal B by photons of energy 4.70 eV is $E_B = (E_A - 1.50)$ eV. If the de-Broglie wavelength of these photoelectrons is $\lambda_B = 2\lambda_A$, then
(a) the work function of A is 2.25 eV
(b) the work function of B is 4.20 eV
(c) E_A = 2.0 eV
(d) E_B = 2.75 eV

20. Photoelectric effect supports quantum nature of light because
(a) there is a minimum frequency of light below which no photoelectrons are emitted.
(b) the maximum kinetic energy of photo electrons depends only on the frequency of light and not on its intensity.
(c) even when the metal surface is faintly illuminated, the photoelectrons leave the surface immediately.
(d) electric charge of the photoelectrons is quantized.

21. An electron and a proton are moving on straight parallel paths with same velocity. They enter a semi-infinite region of uniform magnetic field perpendicular to the velocity. Which of the following statement(s) is/are true?

(a) They will never come out of the magnetic field region.
(b) They will come out travelling along parallel paths.
(c) They will come out at the same time.
(d) They will come out at different times.

22. Assume that the nuclear binding energy per nucleons (B/A) versus mass number (A) is as shown in the figure. Consider a nucleus of $A = 110$. Fission of this nucleus results into 2 fragments.



Which of the following could possibly be the mass number of the resulting nuclei to release energy in fission?

(a) 55 and 55 (b) 70 and 40
(c) 100 and 10 (d) 90 and 20

23. An astronomical refractive telescope has an objective of focal length 20 m and an eyepiece of focal length 2 cm.

(a) The length of the telescope tube is 20.02 m.
(b) The magnification is 1000.
(c) The image formed is inverted.
(d) An objective of a larger aperture will increase the brightness and reduce chromatic aberration of the image.

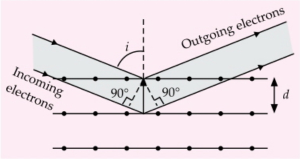
SECTION - 3

Paragraph Type

This section contains 2 paragraphs each describing theory, experiment, data, etc. Four questions relate to two paragraphs with two questions on each paragraph. Each question of a paragraph has only one correct answer among the four choices (a), (b), (c) and (d).

Paragraph for Questions 24 and 25

Wave properties of electrons implies that they will show diffraction effects. Davisson and Germer demonstrated this by diffracting electrons from crystals. The law governing the diffraction from a crystal is obtained by requiring that electron waves reflected from the planes of atoms in a crystal interfere constructively as shown in figure



24. Electrons accelerated by potential V are diffracted from, a crystal. If $d = 1$ Å and $i = 30°$, V should be about (Given, $h = 6.6 \times 10^{-34}$ J s, $m_e = 9.1 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ C)

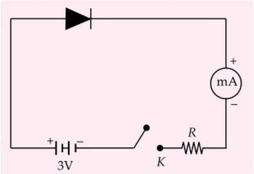
(a) 1000 V (b) 2000 V
(c) 50 V (d) 500 V

25. What is the glancing angle in the above experiment?

(a) 30° (b) 15°
(c) 45° (d) 60°

Paragraph for Questions 26 and 27

When a p-type semiconductor is brought into a close contact with n-type semiconductor, we get a p-n junction with a barrier potential 0.4 V and the width of depletion region is 4×10^{-7} m. This p-n junction is forward biased with a battery of voltage 3 V and negligible internal resistance, in series with a resistor of resistance R, ideal milliammeter and key K as shown in figure. When key K is pressed, a current of 20 mA passes through the diode.



26. The intensity of the electric field in the depletion region when p-n junction is unbiased is

(a) 0.5×10^6 V m^{-1}
(b) 1.0×10^6 V m^{-1}
(c) 2.0×10^6 V m^{-1}
(d) 1.5×10^6 V m^{-1}

27. If an electron with speed 4×10^5 m s^{-1} approaches the p-n junction from the n-side, the speed with which it will enter the p-side is

(a) 1.3×10^5 m s^{-1} (b) 2.7×10^5 m s^{-1}
(c) 1.3×10^6 m s^{-1} (d) 2.7×10^6 m s^{-1}

SECTION - 4

Matching List Type

This section contains 2 multiple choice questions. Each question has matching lists. The codes for the lists have choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

28.

	List-I		List-II
(P)	Ampere Swimming rule	(1)	Direction of induced current in a conductor
(Q)	Fleming's Left hand rule	(2)	Direction of magnetic field lines due to current through circular coil
(R)	Fleming's Right hand rule	(3)	Direction of deflection of magnetic needle due to current in a straight conductor
(S)	Right hand thumb rule	(4)	Direction of force on the current carring conductor due to magnetic field

	P	Q	R	S
(a)	1	2	3	4
(b)	3	1	2	4
(c)	3	4	2	1
(d)	3	4	1	2

29.

	List-I		List-II
(P)	Planck's theory of quanta	(1)	Light energy = $h\nu$
(Q)	Einstein's theory of quanta	(2)	Angular momentum of electron in an orbit
(R)	Bohr's stationary orbit	(3)	Oscillator energies
(S)	de-Broglie waves	(4)	Electron microscope

	P	Q	R	S
(a)	1	2	3	4
(b)	3	1	2	4
(c)	3	4	1	2
(d)	3	4	2	1

SECTION - 5

Assertion-Reason Type

This section contains 4 questions. Read the two statements in the following questions. Of the four choices given, choose the one that best describes the two statements.

(a) Statement-I is true, Statement-II is true; Statement-I is a correct explanation of Statement-I.

(b) Statement-I is true, Statement-II is true; Statement-II is not a correct explanation of Statement-I.

(c) Statement-I is true, Statement-II is false.

(d) Statement-I is false, Statement-II is true.

30. **Statement-I :** For a charged particle moving from point P to point Q, the net work done by an electrostatic field on the particle is independent of the path connecting point P to point Q.

Statement-II : The net work done by a conservative force on an object moving along a closed loop is zero.

31. **Statement-I :** Electric appliances with metallic body, heaters, presses etc., have three pin connections, whereas an electric bulb has a two pin connection.

Statement-II : Three pin connections reduce heating of connecting cables.

32. **Assertion :** In an ac, only capacitor circuit instantaneous power at any instant of time is zero.

Reason : Phase difference between current function and voltage function is 90°.

33. **Assertion :** A particle at rest breaks into two particles of different masses. They fly off in different directions. Their de-Broglie wavelengths will be different.

Reason : Their speed will be different.

SECTION - 6

Integer Value Correct Type

This section contains 5 questions. The answer to each question is a single digit integer, ranging from 0 to 9 (both inclusive).

34. The coercivity of a bar magnet is 120 A m^{-1}. It is to be demagnetised by placing it inside a solenoid of length 120 cm and number of turns 72. The current (in A) flowing through the solenoid is

35. An ac generator gives an output voltage of $E = 170 \sin 56.52t$. What is the frequency of alternating voltage produced (in hertz)?

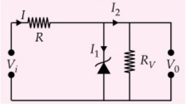
36. A person with two normal eyes wants to see full width of his face by a plane mirror. The eye to eye and ear to ear distances of his face are 10 cm and 14 cm respectively. The minimum width (in cm) of required mirror is

37. A proton is fired from very far away towards a nucleus with charge $Q = 120\ e$, where e is the electronic charge. It makes a closest approach of 10 fm to the nucleus. The de Broglie wavelength (in units of fm) of the proton at its start is

(Take the proton mass, $m_p = (5/3) \times 10^{-27}$ kg

$h/e = 4.2 \times 10^{-15}$ J s C^{-1}, $\dfrac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ m F^{-1}, 1 fm = 10^{-15} m)

38. A 10 V Zener diode along with a series resistance R is connected across a 40 V supply as shown in figure. If the maximum value Zener current is 50 mA, the minimum value of resistance R is $a \times 10^2\ \Omega$. Find the integer value of a.



SOLUTIONS

1. **(c) :** Number of electrons,

$$n = \frac{6\times10^{23}}{63.5}\times10\times\frac{1}{10^6} = \frac{6\times10^{18}}{63.5}$$

$$q = \frac{6\times10^{18}\times1.6\times10^{-19}}{63.5}$$

or $q = 1.5\times10^{-2}\,\text{C}$

$$F = \frac{9\times10^{9}\times1.5\times10^{-2}\times1.5\times10^{-2}}{\left(\frac{10}{100}\right)}$$

$$= 2.0\times10^{8}\ \text{N}$$

2. **(a) :** As, $\tau = MB\sin\theta$

where, M is magnetic dipole moment, B the magnetic field and θ is the angle between them

Given, $\tau_1 = 10^{-5}$ N m, $\theta_1 = 90°$, $\theta_2 = 30°$.

$$\tau_1 = MB\sin 90° \quad ...(i)$$

$$\tau_2 = MB\sin 30° \quad ...(ii)$$

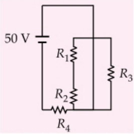
Dividing eqn. (i) by eqn. (ii), we get

$$\frac{\tau_1}{\tau_2} = \frac{10^{-5}}{\tau_2} = \frac{1}{1/2}$$

$$\Rightarrow \quad \tau_2 = \frac{10^{-5}}{2} = \frac{10}{2}\times10^{-6}$$

$$= 5\times10^{-6}\ \text{N m}$$

3. **(c) :** Total resistance of the circuit



$$R = R_4 + \frac{(R_1+R_2)\times R_3}{(R_1+R_2)+R_3}$$

$$= 35 + \frac{(15+15)\times30}{(15+15)+30}$$

$$= 35 + \frac{30\times30}{30+30} = 50\ \Omega$$

Current in circuit, $I = \dfrac{50}{50} = 1\,\text{A}$

Current through R_3, $I' = I_2 = \dfrac{1}{2}$ V

Potential difference across R_3

$$= I'\times R_3 = \frac{1}{2}\times30 = 15\ \text{V}$$

4. **(c) :** Frequency of sound = 600 Hz inside water.

Frequency of a wave depends on the source and it does not change during refraction when wave travels from one medium to another.

$\therefore$ Frequency heard in air = 600 Hz.

5. **(c) :** By energy conservation,

loss in kinetic energy = gain in potential energy

$$\frac{1}{4\pi\varepsilon_0}\frac{(Ze)(2e)}{r_{\min}} = 5\times1.6\times10^{-13}$$

$$\therefore\ r_{\min} = \frac{1}{4\pi\varepsilon_0}\frac{2Ze^2}{5\times1.6\times10^{-13}}$$

$$\text{or } r_{\min} = \frac{9\times10^9\times2\times92\times(1.6\times10^{-19})^2}{5\times1.6\times10^{-13}}$$

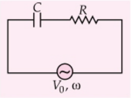
$$= 5.3\times10^{-14}\ \text{m}$$

$$\therefore\ r_{\min} = 5.3\times10^{-12}\ \text{cm}$$

The distance of closest approach is of the order of 10^{-12} cm.

6. **(b):** Impedance of the circuit,

$$Z = \sqrt{(X_C)^2 + (R)^2} = \sqrt{\left(\frac{1}{\omega C}\right)^2 + (R)^2}$$



As ω increases, Z decreases. Current in the circuit, $I = \dfrac{V_0}{Z}$

When ω is increased, the impedance of the circuit decreases and the current through the bulb increases. Therefore the bulb glows brighter.

7. **(d):** In a perpendicular magnetic field,

Magnetic force = centripetal force

$$i.e.,\ Bqv = \frac{mv^2}{r} \text{ or } r = \frac{mv}{Bq}$$

$$\therefore\ \frac{r_1}{r_2} = \frac{v_1}{v_2}\times\frac{B_2}{B_1}$$

$$\frac{r_1}{r_2} = \frac{1}{2}\times\frac{1}{2}$$

$$r_2 = 4r_1 = 4r$$

8. **(d):** The electron placed in the path of electromagnetic wave will experience force due to electric field vector and not due to magnetic field vector.

9. **(d):** The focal length of the lens

$$\frac{1}{f} = (\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\frac{1}{f} = (1.5-1)\left(\frac{1}{20}-\frac{1}{30}\right)$$

$$\frac{1}{f}=0.5\left(\frac{30-20}{600}\right) \text{ or } \frac{1}{f}=\frac{1}{2}\times\frac{10}{600}=\frac{1}{120}$$

$\therefore\ f = 120$ cm

10. (a) : Given, $h = 150$ m, $R = 6.4 \times 10^6$ m

Average population density $= 10^3$ km^{-2}

$= (10^3)(10^3)$ m$^{-2} = 10^{-3}$ m^{-2}

Distance upto which the transmission could be viewed

$$d=\sqrt{2hR}$$

Total area over which transmission could be viewer

$= \pi d^2 = 2\pi hR$

Population covered $= 10^{-3} \times 2\pi hR$

$= 10^{-3} \times 2 \times 3.14 \times 150 \times 6.4 \times 10^6$

$= 60.29$ lakh

11. (c) **12. (a)**

13. (b): The impedance of the circuit is

$$Z=\sqrt{R^2+(X_L-X_C)^2}$$

Here,

$$X_L = \omega L = 400 \times 20 \times 10^{-3} = 8 \text{ H}$$

$$X_C=\frac{1}{\omega C}=\frac{1}{400\times 625\times 10^{-6}}=4 \text{ F}$$

$$\therefore\quad Z=\sqrt{(3)^2+(8-4)^2}=5$$

$$I=\frac{V}{Z}=\frac{300}{5}=60 \text{ A}$$

14. (a) : The wavelengths of the spectral lines in the Balmer series is given by

$$\frac{1}{\lambda}=RZ^2\left[\frac{1}{2^2}-\frac{1}{n^2}\right] \quad n=3,4,5,6\,.....$$

For hydrogen atom, $Z = 1$, $n = 3$ for first spectral line

$$\therefore\ \frac{1}{\lambda_H}=R(1)^2\left[\frac{1}{2^2}-\frac{1}{3^2}\right]=R\left[\frac{1}{4}-\frac{1}{9}\right]=R\left[\frac{5}{36}\right] \quad ..(i)$$

For He$^+$ ion, $Z = 2$, $n = 4$ for second spectral line

$$\therefore\ \frac{1}{\lambda_{He^+}}=R(2)^2\left[\frac{1}{2^2}-\frac{1}{4^2}\right]=4R\left[\frac{1}{4}-\frac{1}{16}\right]$$

$$=4R\left[\frac{3}{16}\right] \quad ...(ii)$$

Divide (i) by (ii), we get

$$\frac{\lambda_{He^+}}{\lambda_H}=\frac{5}{27}$$

$$\lambda_{He^+}=\lambda_H\times\frac{5}{27}=(6561\text{ Å})\times\frac{5}{27}=1215\text{ Å}$$

15. (b, c) : D_1 is forward biased and D_2 reverse biased. Therefore, current through the resistance and D_1

i.e., $I=\frac{V}{R}=\frac{5}{4}=1.25$ A

16. (b, c) : In order to increase the range of ammeter, a low resistance is connected in parallel with the ammeter. Let S be the low resistance

$$\therefore\ \frac{S}{G}=\frac{I_g}{I-I_g} \text{ or } S=G\frac{I_g}{I-I_g}$$

$$\text{or } S=(100)\frac{50\times10^{-6}}{(5\times10^{-3})-(50\times10^{-6})}$$

$(\because\ 5 \text{ mA} >> 50\ \mu\text{A})$

$$\text{or } S=\frac{100\times50\times10^{-6}}{5\times10^{-3}} \text{ or } S=1\ \Omega.$$

Option (c) is correct.

In order to change ammeter into a voltmeter, a high resistance is connected in series with the ammeter. Let the high resistance be R.

$$\therefore\quad I_g(R+G)=V$$

$$\text{or } R=\frac{V}{I_g}-G \text{ or } R=\frac{10}{50\times10^{-6}}-100$$

$$\text{or } R=200\text{ k}\Omega-100\,\Omega \text{ or } R\approx 200\text{ k}\Omega.$$

Option (b) is correct.

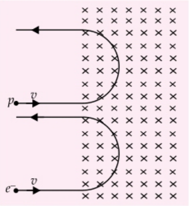
17. (c, d) : As $\mu\geq\frac{1}{\sin C}>\frac{1}{\sin 45^\circ}>\frac{1}{1/\sqrt{2}}\geq\sqrt{2}=1.414.$

$\therefore$ Possible value of μ are 1.5 and 1.6.

18. (a, b, c, d) **19. (a, b, c)**

20. (a, b, c)

21. (b, d) :



From figure, it is clear that they will come out travelling along parallel paths. Hence, (b) is correct.

The velocity of the charged particle is normal to the magnetic field and the magnetic field is uniform, so the particle will follow a circular path. The radius of the circular path is

$$R=\frac{mv}{qB}$$

For proton, $R_p = \frac{m_p v}{eB}$; For electron, $R_e = \frac{m_e v}{eB}$

$m_p > m_e \Rightarrow R_p > R_e$

Time spent by the particle in the field is $T = \frac{\pi m}{qB}$

$\therefore$ For electron, $T_e = \frac{\pi m_e}{eB}$; For proton, $T_p = \frac{\pi m_p}{eB}$

$\because m_p > m_e \Rightarrow T_p > T_e$

Hence (d) is correct.

22. (a, b) : To release energy in a nuclear process, binding energy per nucleon should increase.

23. (a, b, c) :

(a) Length of the telescope,

$L = f_0 + f_e = 20 \text{ m} + 0.02 \text{ m} = 20.02 \text{ m}$

(b) Magnification, $\mu = \frac{f_0}{f_e} = \frac{20 \text{ m}}{0.02 \text{ m}} = 1000$

(c) In an astronomical telescope, image formed is inverted.

24. (c)

25. (d): Glancing angle is the inclination of the incident rays with the plane of the atoms which is,

$$\theta = 90° - 30° = 60°.$$

26. (b): As $E = \frac{V_B}{d} = \frac{0.4}{4 \times 10^{-7}} = 1 \times 10^6 \text{ V m}^{-1}$

27. (a) : Let v_1 be the speed of electron when enters the depletion region and v_2 is the speed when it comes out of depletion layer. According to principle of conservation of total energy,

K.E. of the incident electron = workdone against potential barrier + K.E. of the emerging electron.

i.e. $\frac{1}{2} m v_1^2 = eV_B + \frac{1}{2} m v_2^2$

or $\frac{1}{2} \times (9.1 \times 10^{-31}) \times (4 \times 10^5)^2$

$= (1.6 \times 10^{-19} \times 0.4) + \frac{1}{2} \times 9.1 \times 10^{-31} \times v_2^2$

On solving, we get, $v_2 = 1.3 \times 10^5 \text{ m s}^{-1}$.

28. (d): Ampere swimming rule tells the direction of deflection of the magnetic needle due to current in a straight conductor, when magnetic needle is held below the conductor.

Fleming's left hand rule tells the direction of force on the current carrying conductor placed perpendicular to a uniform magnetic field.

Fleming's right hand rule tells the direction of induced current.

Right hand thumb rule tells the direction of magnetic field lines due to current through circular coil.

29. (b): Planck's theory of quanta is for quantisation of energies of oscillations.

Einstein theory of quanta gave the energy of photon as $h\upsilon$.

Bohr's stationary orbit in an atom is one in which the angular momentum of electron is integral multiple of $h/2\pi$.

de-Broglie waves are used in the working of electron microscope.

30. (a) : As electrostatic field is conservative,

$W = q\,(V_Q - V_P)$,

i.e., work done depends only on end positions and not on the path followed.

31. (c) : The electrical appliances with metallic body like heater, press etc. have three pin connections. Two pins are for supply line and third pin is for earth connection for safety purposes. Here statement-II is wrong.

32. (d) : If $V = V_0 \sin \omega t$, then current function will lead the voltage function by 90°

$\therefore$ $I = I_0 \cos \omega t$

$\therefore$ Instantaneous power

$P = VI = V_0 I_0 \sin \omega t \cos \omega t$

or $P \neq 0$ at all times.

33. (d): As $\lambda = \frac{h}{p}$

Their momenta are same. Hence their wavelengths are same.

34. (2)

35. (9) : Here $E = 170 \sin 56.52\, t$

Compare with the standard from of equation of alternating emf,

$E = E_0 \sin \omega t$, $\omega = 56.52$,

$2\,\pi\upsilon = 56.52$

$$\upsilon = \frac{56.52}{2\pi} = \frac{56.52}{2 \times 3.14} = 9 \text{ Hz}$$

36. (2) : Here, eye to eye distance, $a = 8$ cm

ear to ear distance, $d = 12$ cm

$\therefore$ Minimum width of required mirror is

$$x = \frac{b-a}{2} = \frac{12-8}{2} = 2 \text{ cm}.$$

37. (7)

38. (6) : Here, $V_i = 40$ V, $I_1 = 50$ mA

Since maximum current flows through Zener, hence, $I_2 = 0$

$\therefore$ Maximum current, $I = I_1 + I_2 = 50 \text{ mA} + 0$

$= 50 \text{ mA} = 50 \times 10^{-3}$ A

Voltage drop across Zener, $V_0 = 10$ V

Since I is maximum, hence maximum value of R is

$$R = \frac{V_i - V_0}{I} = \frac{40-10}{50 \times 10^{-3}} = 6 \times 10^2 \Omega$$

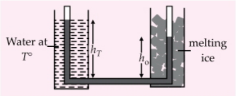
$\therefore$ $a = 6$.

Thought Provoking Problems

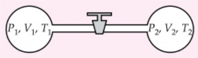
THERMODYNAMICS

By : Prof. Rajinder Singh Randhawa*

1. In figure shown, left arm of a U-tube is immersed in a hot water bath at temperature T, and right arm is immersed in a bath of melting ice, the height of manometric liquid in respective columns is h_T and h_0. Determine the coefficient of expansion of the liquid.



2. Two thermally insulated vessels, filled with air are connected by a short tube containing a valve, initially closed. The pressures, volumes and temperatures in the two vessels are P_1, V_1, T_1 and P_2, V_2, T_2 respectively. Find the P, T values after opening the valve.



3. A capillary tube of constant cross-sectional area is filled with an ideal gas. The temperature of the gas varies linearly from one end ($x = 0$) to the other ($x = L$), according to the equation

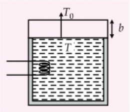
$$T = T_0 + \left(\frac{T_L - T_0}{L}\right)x.$$

The volume of the capillary is V and the pressure P is uniform throughout. Determine the number of moles of the gas in the capillary.

4. An ideal gas of molecular weight M is contained in a tall vertical cylindrical vessel of base area A and height h. The temperature of the gas is T and assumed to be constant. In uniform gravitational field, find the height at which the centre of gravity of the gas is located.

5. An ideal gas of molar mass M is filled in a horizontal cylinder closed at one end. The cylinder rotates with a constant angular velocity ω about a vertical axis passing through the open end of the cylinder. The pressure at the axis of the cylinder is P_0 and temperature T. Find the air pressure as a function of the distance r from the rotation axis, in isothermal condition.

6. A cylinder with adiabatic walls contains water. The cylinder is closed on top with a metal cover of area A, thickness b and thermal conductivity K. Heat capacity of water is C, temperature of surroundings is T_0 assumed to be constant. An electric heater produces heat at a rate H. Find the temperature of water as a function of time after the heater is switched on.



7. Consider one mole of an ideal gas whose volume changes with temperature as $V = \dfrac{\alpha}{T}$, where α is constant. Find the amount of heat required to raise its temperature by ΔT, if its adiabatic constant is γ.

SOLUTIONS

1. Since the liquid is in hydrostatic equilibrium,

$$\rho_T g h_T = \rho_0 g h_0, \Rightarrow \rho_T = \frac{\rho_0 h_0}{h_T}$$

Also, $V_T = V_0(1 + \gamma T)$. ...(i)

and $\rho_T V_T = \rho_0 V_0 \Rightarrow \rho_T = \dfrac{\rho_0}{(1+\gamma T)}$ ...(ii)

From (i) and (ii), we get

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$h_T = h_0(1 + \gamma T)$.
which on solving for γ, we get

$$\gamma = \frac{(h_T - h_0)}{h_0 T}$$

2. In equilibrium, both the vessels have the same pressure.
After mixing of air, total no. of moles, $n = n_1 + n_2$.
The total volume of the air, $V = V_1 + V_2$.
Let the common temperature attained be T.

$\therefore \quad P(V_1 + V_2) = (n_1 + n_2)RT$

or $$P = \frac{(n_1 + n_2)RT}{V_1 + V_2} \quad \ldots\text{(i)}$$

The combined system is thermally insulated, hence, $Q = 0$
System does no mechanical work, since $dV = 0$,
i.e., $dW = PdV = 0$.
From 1st law of thermodynamics, $dQ = dU + dW$.
Hence, $dU = 0$.
There is no change in internal energy.
$\therefore$ The internal energy U of an ideal gas is given by

$$U = nC_V T = \frac{nRT}{\gamma - 1} = \frac{PV}{\gamma - 1} \qquad \left[\because\ C_V = \frac{R}{\gamma - 1}\right]$$

$$U_{\text{initial}} = \frac{n_1 R T_1}{\gamma - 1} + \frac{n_2 R T_2}{\gamma - 1} = \frac{R}{\gamma - 1}(n_1 T_1 + n_2 T_2) \quad \ldots\text{(ii)}$$

$$U_{\text{final}} = \frac{(n_1 + n_2)RT}{\gamma - 1} \quad \ldots\text{(iii)}$$

$\because \quad U_{\text{initial}} = U_{\text{final}}$

$\Rightarrow \quad n_1 T_1 + n_2 T_2 = (n_1 + n_2)T$

$$\Rightarrow \quad T = \left(\frac{n_1 T_1 + n_2 T_2}{n_1 + n_2}\right) \quad \ldots\text{(iv)}$$

or $$T = \frac{P_1 V_1 + P_2 V_2}{\left[\left(\frac{P_1 V_1}{T_1}\right) + \left(\frac{P_2 V_2}{T_2}\right)\right]} \qquad \left(\because\ n = \frac{PV}{RT}\right)$$

or $$T = \frac{T_1 T_2 (P_1 V_1 + P_2 V_2)}{(P_1 V_1 T_2 + P_2 V_2 T_1)}$$

Now, $$P = \frac{(n_1 + n_2)RT}{V_1 + V_2} = \frac{R}{V_1 + V_2}(n_1 + n_2)T$$

From equation (iv), $(n_1 + n_2)T = n_1 T_1 + n_2 T_2$

$$\therefore \quad P = \frac{R}{(V_1 + V_2)}(n_1 T_1 + n_2 T_2) = \frac{P_1 V_1 + P_2 V_2}{V_1 + V_2}$$

(as $P_1 V_1 = n_1 R T_1$ and $P_2 V_2 = n_2 R T_2$)

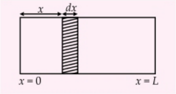
3. Consider an infinitesimal length dx of tube at a distance x from one end.

$\therefore$ Volume of this section $= Adx$

Here, $T = T_0 + \left(\frac{T_L - T_0}{L}\right)x$

Volume of the capillary,

$V = AL$, is constant.



Applying ideal gas equation to this differential volume, we get

$$P(Adx) = dnRT = dnR\left[T_0 + \left(\frac{T_L - T_0}{L}\right)x\right].$$

or $$\int_0^L \frac{dx}{\left[T_0 + \left(\frac{T_L - T_0}{L}\right)x\right]} = \int_0^n \frac{R}{PA} dn$$

$$\frac{L}{(T_L - T_0)} \ln\left[T_0 + \left(\frac{T_L - T_0}{L}\right)x\right]_0^L = \frac{nR}{PA},$$

Solving, we get

$$n = \frac{P(AL)}{(T_L - T_0)R} \ln\left(\frac{T_L}{T_0}\right) = \frac{PV}{(T_L - T_0)R} \ln\left(\frac{T_L}{T_0}\right).$$

4. Consider a layer of thickness dh as shown in figure.
Volume of this layer $= dV = Adh$
Mass of this layer

$$dm = \rho dV = \rho(Adh) \quad \ldots\text{(i)}$$

As, density at a height h is given by

$$\rho = \rho_0 e^{-Mgh/RT}$$

so equation (i), becomes

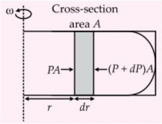
$$dm = \rho_0 A e^{-Mgh/RT} \cdot dh.$$

The height of centre of gravity is

$$h_{C.G.} = \frac{\int_0^\infty (gdm) \cdot h}{\int_0^\infty gdm} = \frac{\int_0^\infty \rho_0 g A h e^{-Mgh/RT} dh}{\int_0^\infty \rho_0 g A e^{\frac{-Mgh}{RT}} dh}$$

$$\therefore \quad h_{C.G.} = \frac{RT}{Mg}$$

5. Consider a small area element of thickness dr, at a distance r from the axis.



$$(P + dP)A - PA = r\omega^2 dm$$

or $\quad (dP)A = r\omega^2(\rho A dr)$

$\therefore \quad dP = \rho\omega^2 r dr \ \ldots$(i)

From ideal gas equation,

$$\rho = \frac{PM}{RT} \qquad \ldots\text{(ii)}$$

From equations (i) and (ii), we get

$$dP = \frac{PM}{RT}\omega^2 r\, dr \quad \text{or} \quad \frac{dP}{P} = \frac{M}{RT}\omega^2 r dr$$

Integrating both sides, we get

$$\int_{P_0}^{P} \frac{dP}{P} = \frac{M\omega^2}{RT}\int_0^r r\, dr \quad \text{or} \quad \ln\frac{P}{P_0} = \frac{M\omega^2}{RT}\frac{r^2}{2}$$

$$\therefore \quad P = P_0 e^{M\omega^2 r^2/2RT}$$

6. Heat conducted through metallic slab,

$$\frac{dQ'}{dt} = \frac{KA(T - T_0)}{b}.$$

Heat gain by water through heater = H.
Net rate at which heat is gained by water,

$$\frac{dQ'}{dt} = H - \frac{KA}{b}(T - T_0)$$

$$\Rightarrow \quad C\frac{dT}{dt} = H - \frac{KA}{b}(T - T_0)$$

$$\int_{T_0}^{T} \frac{CdT}{\left[H - \frac{KA}{b}(T - T_0)\right]} = \int_0^t dt$$

or $\quad C\left(\dfrac{-b}{AK}\right)\left[\ln\left\{H - \dfrac{AK(T - T_0)}{b}\right\}\right]_{T_0}^{T} = t$

or $\quad \dfrac{-bC}{AK}\ln\left[\dfrac{H - \frac{AK}{b}(T - T_0)}{H}\right] = t$

$$H - \frac{AK(T - T_0)}{b} = He^{-\left(\frac{AKt}{bC}\right)}$$

Solving, we get,

$$T = T_0 + \frac{bH}{AK}\left[1 - e^{-\left(A\frac{K}{bc}\right)t}\right]$$

7. Since $PV = nRT$,

or $\quad PV = nR\dfrac{\alpha}{V} \qquad \left(\because\ T = \dfrac{\alpha}{V}\right)$

or $\quad P = \dfrac{nR\alpha}{V^2}$

The work done in this process

$$W = \int_{V_i}^{V_f} P \cdot dV = \int_{V_i}^{V_f} \frac{nR\alpha}{V^2} dV = nR\alpha\left(\frac{1}{V_i} - \frac{1}{V_f}\right)$$

$$\therefore \quad W = nR(T_i - T_f) = -\,nR(T_f - T_i) = -\,nR\Delta T$$

Since $T_i < T_f$, W is negative,
i.e. work is done on the system.
Again, change in internal energy,

$$\Delta U = \frac{nR\Delta T}{\gamma - 1}$$

$\therefore \quad$ Amount of heat required, $\Delta Q = \Delta U + \Delta W$

$$= \frac{nR\Delta T}{\gamma - 1} - nR\Delta T = \frac{nR\Delta T}{\gamma - 1}[2 - \gamma]$$

YOU ASKED WE ANSWERED

Do you have a question that you just can't get answered?
Use the vast expertise of our mtg team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough.
The best questions and their solutions will be printed in this column each month.

Q1. It is a common observation that as a lightbulb ages, it gives off less light than when new. Why?

(Ankit, Moradabad, U.P.)

Ans. There are two reasons for this, one electrical and one optical, but both are related to the same phenomenon occurring within the bulb. The filament of lightbulb is made of a tungsten wire that, in an old lightbulb, has been kept at a high temperature for many hours. These high temperatures cause tungsten to be evaporated from the filament, decreasing its radius. From $R = \rho\, l/A$, we see that a decreased cross-sectional area leads to an increase in resistance of the filament. This increasing resistance with age means that the filament will carry less current for the same applied voltage. With less current in the filament, there is less light output, and the filament glows more dimly.
At the high operating temperature of the filament, tungsten atoms leave the surface of the filament, such as water molecules evaporate from a puddle of water. These atoms are carried away by convection currents in the gas in the bulb and are deposited on the inner surface of the glass. With time, the glass becomes less transparent because of this tungsten coating, which decreases the amount of light that passes through the glass.

Q2. The launch area for the European Space Agency is not in Europe it is in South America. Why?

(Rohit Garg, New Delhi)

Ans. Placing a satellite in Earth orbit requires, providing a large tangential speed to the satellite, which is the task of the rocket propulsion system. Anything that reduces the requirements on the propulsion system is a welcome contribution. The surface of the Earth is already travelling toward the east at a high speed, due to the rotation of the Earth. Thus, if rockets are launched toward the east, the rotation of the Earth provides some initial tangential speed, reducing somewhat the requirements on the propulsion system. If rockets were launched from Europe, which is at a relatively large latitude, the contribution of the Earth's rotation is relatively small because the distance between Europe and the rotation axis of the Earth is relatively small. The ideal place for launching is at the equator, which is as far as one can be from the rotation axis of the Earth and still be on the surface of the Earth. This results in the largest possible tangential speed due to the Earth's rotation. The European Space Agency exploits this advantage by launching from French Guiana, which is only a few degrees north of the equator.
A second advantage of this launch location is that launching toward the east takes the spacecraft over water. In the event of an accident or a failure, the wreckage will fall into the ocean rather than into populated areas as it would if launched to the east from Europe. This is a primary reason why the United States launches spacecraft from Florida rather than California, despite the more favourable weather conditions in California.

Q3. If you sit in front of a fire with your eyes closed, you can feel significant warmth in your eyelids. If you now put on a pair of eyeglasses and repeat this activity, your eyelids will not feel nearly so warm. Why?

(Rishabh, Chennai, Tamil Nadu)

Ans. Much of the warmth you feel is due to electromagnetic radiation from the fire. A large fraction of this radiation is in the infrared part of the electromagnetic spectrum. Your eyelids are particularly sensitive to infrared radiation. On the other hand, glass is very opaque to infrared radiation. Thus, when you put on the glasses, you block much of the radiation from reaching your eyelids, and they feel cooler.

Q4. When is the rate at which energy is being delivered to a lightbulb higher — just after it is turned on, and the glow of the filament is increasing, or after it has been on for a few seconds and the glow is steady?

(Shubham, Kolkata, W.B.)

Ans. As the voltage is applied across the cold filament when the lightbulb is first turned on, the resistance of the filament is low. Thus, the current is high, and a relatively large amount of energy is delivered to the bulb per unit time. As the filament warms up, its resistance rises, and the current falls. As a result, the rate of energy delivered to the bulb falls. The large current spike at the beginning of operation is the reason that lightbulbs often fail just as they are turned on.

Q5. Is it possible to cool your kitchen by leaving the refrigerator door open or cool your bedroom by putting a window air conditioner on the floor, near by the bed?

(Reena Patel, Surat, Gujarat)

Ans. Whatever heat Q_C is removed from the air directly in front of the open refrigerator is deposited directly back into the kitchen at the rear of the unit. Moreover, according to the second law, work W is needed to move that heat from cold to hot, and the energy from this work is also deposited into the kitchen as additional heat. Thus, the open refrigerator puts into the kitchen an amount of heat $Q_H = Q_C + W$, which is more than it removes. Rather than cooling the kitchen, the open refrigerator warms it up. Putting a window air conditioner on the floor to cool your bedroom is similarly a no-win game. The heat pumped out the back of the air conditioner and into the bedroom is greater than the heat pulled into the front of the unit. Consequently, the air conditioner actually warms the bedroom.

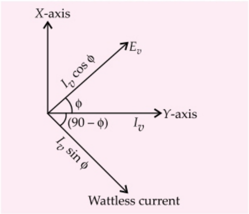
Q6. How can alpha, beta and gamma rays be detected using our modern technology?

(G.S. Sudarshan, Mysore, Karnataka)

Ans. We know that alpha particle decay decreases the mass number by 4 and atomic number by 2. The beta-particle increases the atomic number by 1. Gamma rays are highly energetic photons. Alpha particles which are doubly charged helium nuclei are always repelled when directed towards heavy nuclei *e.g.*, gold as used in Rutherford's experiment. In modern technology, these repelled alpha rays are made to fall on radioactive substances kept in sealed chambers which produce some interaction like heat, light etc., which can run an electric circuit with LEDs. The beta particles, which are negatively charged like electrons when released into a magnetic field directed perpendicularly trace a circular path. The beta particles when made to rotate in sealed chambers coated with electroluminescent material show a glow along the path of beta particles. The gamma rays which are just highly energetic photons can be detected by the interaction of the photons with the electrons of any crystal. The photons on collision with the electrons in the crystal undergo a shift in their energy after collision. This shift in the wavelength of photon is the famous compton shift. This can be detected by photo detectors.

Q7. Current is a scalar quantity then why the vector component of current are resolved to find the wattless current?

(Satish Sheoran, Bhiwani, Haryana)



Ans. Current is a scalar quantity as it does not obey the parallelogram law of addition of vectors, which is an essential condition for any physical quantity to be a vector.

The use of vectors in the form of phasor diagrams for studying ac circuits is only better illustration of the properties like phase, amplitude, leading or lagging of sinusoidal signals whose amplitude represents the length of the vector and phase by its orientation with respect to the origin.

Q8. What is the difference between vapour and gases?

(Anshul Parmar, Hamirpur, H.P.)

Ans. Gases are molecules of elements which obey the ideal gas equation in the case of ideal gases or with certain modifications in the ideal gas equation for non-ideal gases. Also, they have two specific heats, one at constant pressure C_p and other at constant volume C_v.

Vapour on the other hand can be formed from any element or compound by heating it up to the temperature of latent vaporisation. Generally, the vapours are characterized by the properties like specific heat of their phase which exists at normal temperature and pressure. Their thermodynamics is not studied using the gas equation.

Solution Senders of Physics Musing

SET-6

1. Shruti (M.P.)
2. Mukesh (U.P.)
3. Arun Kumar (Kerala)
4. Shankar Gupta (Bihar)

PHYSICS MUSING

SOLUTION SET-6

1. If ρ is the density of the material of the carpet, initial mass of the carpet (cylinder) $M = \pi R^2 L\rho$ and when its radius becomes half the mass of cylindrical part will be

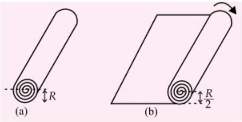
$$M_F = \pi\left(\frac{R}{2}\right)^2 L\rho = \frac{M}{4}$$

So initial potential energy of the carpet $U_I = MgR$ while final potential energy.

$$U_F = \frac{M}{4}g\left(\frac{R}{2}\right) = \frac{MgR}{8}$$

So loss in potential energy due to unrolling radius changes from R to $R/2$

$$\Delta U = U_I = U_F = MgR - \frac{1}{8}MgR = \frac{7}{8}MgR \quad \ldots\text{(i)}$$



This loss in potential energy is equal to increase in rotational kinetic energy which is

$$K = K_T + K_R = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

If v is the velocity when half the carpet has unrolled

$$\therefore \quad v = \frac{R}{2}\omega,\ M \to \frac{M}{4} \text{ and } I = \frac{1}{2}\left[\frac{M}{4}\right]\left[\frac{R}{2}\right]^2 = \frac{MR^2}{32}$$

$$\therefore \quad K = \frac{1}{2}\left[\frac{M}{4}\right]v^2 + \frac{1}{2}\left[\frac{MR^2}{32}\right]\left[\frac{2v}{R}\right]^2$$

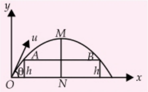
$$\Rightarrow \quad K = \frac{1}{8}Mv^2 + \frac{1}{16}Mv^2 = \frac{3}{16}Mv^2 \quad \ldots\text{(ii)}$$

So from equation (i) and (ii)

$$\frac{3}{16}Mv^2 = \frac{7}{8}MgR$$

$$\therefore \quad v = \sqrt{\frac{14gR}{3}}$$

2. The situation is shown in figure. Let θ be the angle of projection and u the velocity of projection.



Maximum height $MN = 2h$

$$MN = 2h = \frac{u^2\sin^2\theta}{2g}$$

$$\therefore \quad u\sin\theta = 2\sqrt{gh} \quad \ldots\text{(i)}$$

Let t be the time taken by stone to attain the vertical height h above the ground.

$$\therefore \quad h = (u\sin\theta)t - \frac{1}{2}gt^2$$

$$\Rightarrow \quad t^2 - \left(\frac{2u\sin\theta}{g}\right)t + \frac{2h}{g} = 0$$

$$t = \frac{u\sin\theta}{g} \pm \sqrt{\frac{u^2\sin^2\theta}{g^2} - \frac{2h}{g}}$$

Substituting the value of $u\sin\theta$ from (i),

$$t = \frac{2\sqrt{gh}}{g} \pm \sqrt{\frac{4gh}{g^2} - \frac{2h}{g}} = \sqrt{\frac{4h}{g}} \pm \sqrt{\frac{2h}{g}}$$

$$\Rightarrow \quad t_1 = \sqrt{\frac{4h}{g}} - \sqrt{\frac{2h}{g}},\ t_2 = \sqrt{\frac{4h}{g}} + \sqrt{\frac{2h}{g}}$$

where t_1 and t_2 are time to reach A and B respectively shown in the figure. If v is the horizontal velocity of bird, then

$AB = vt_2$.

AB is also equal to $u\cos\theta(t_2 - t_1)$, where $u\cos\theta$ is constant horizontal velocity of stone

$$t_2 - t_1 = 2\sqrt{\frac{2h}{g}}$$

$$\therefore \quad u\cos\theta \times 2\sqrt{\frac{2h}{g}} = vt_2$$

$$\frac{v}{u\cos\theta} = \frac{2\sqrt{\frac{2h}{g}}}{t_2} = \frac{2\sqrt{\frac{2h}{g}}}{\sqrt{\frac{2h}{g}}(\sqrt{2}+1)}$$

$$= \frac{2}{\sqrt{2}+1} = 2(\sqrt{2}-1)$$

3. Consider dN number of turns of radius r and thickness dr. Let dE be the corresponding induced emf, then

$$dE = (dN)\left(\frac{d\phi}{dt}\right)$$

$$dE = dN\ \frac{d}{dt} \times (\pi r^2 \times B_0 \sin\omega t)$$

$$dE = \left(\frac{N}{a}\right)dr\ \ (\pi r^2 w \times B_0 \cos\omega t)$$

$$E = \int dE = \left(\frac{N\pi\omega B_0\cos\omega t}{a}\right)\int_0^a r^2 dr$$

$$= \frac{N\pi\omega(B_0 \cos\omega t)a^3}{3a} = \frac{N\pi\omega B_0 \cos\omega t a^2}{3} \qquad ...(i)$$

$$E_{max} = \frac{\pi N a^2 B_0 \omega}{3}. \qquad ...(ii)$$

comparing (ii) with (i) we get, $n = 3$.

4. Given, the pitch of screw gauge = 1 mm and total number of division on the ciruclar scale = 50

Least Count, $L.C = \frac{1\text{ mm}}{50} = 0.02$ mm

The instruments has a positive zero error

$e = n \times L.C$

$= 6 \times 0.02 = 0.12$ mm

i.e. zero correction = – 0.12 mm

Linear scale reading (LSR) = 3 × 1 mm = 3 mm

Circular scale reading (CSR) = 31 × (0.02 mm)

= 0.62 mm

Measured reading = $LSR + CSR$

= 3 + 0.62 = 3.62 mm

True reading = 3.62 – 0.12 = 3.50 mm

5. **(c) :** $\sum W_{net} = \Delta K$

As particle is moving slowly, this means $\Delta K = 0$

$\Rightarrow \quad W_N + W_F + W_{mg} = \Delta K$

but $W_N = 0$, as $\vec{N} \perp d\vec{r}$

$\Rightarrow \quad 0 + W_F - mgh = 0 \quad \Rightarrow \quad W_F = mgh$

6. **(c) :** Since, kinetic energy $K = \frac{1}{2}m\omega^2 A^2 \cos^2 \omega t$

and potential energy, $U = \frac{1}{2}m\omega^2 A^2 \sin^2 \omega t$

or $\quad K - U = \frac{1}{2}m\omega^2 A^2[\cos^2 \omega t - \sin^2 \omega t]$

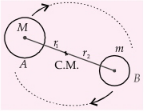
$$= \frac{1}{2}m\omega^2 A^2 \cos 2\omega t$$

$\therefore$ Angular frequency = 2ω,

time period, $= \frac{2\pi}{2\omega} = \frac{\pi}{\omega} = \frac{\pi \times T}{2\pi}\left[\because \omega = \frac{2\pi}{T}\right]$

$$= \frac{\pi \times 4}{2\pi} = 2 \text{ s}$$

7. **(a) :** Moment of inertia of solid sphere of mass M and radius R about an axis passing through the centre of mass is $I = \frac{2}{5}MR^2$. Let the radius of disc be r. Moment of inertia of circular disc of radius r and mass M about an axis passing through the centre of mass and perpendicular to its plane $= \frac{1}{2}Mr^2$



$\therefore$ Using theorem of parallel axes, moment of inertia of disc about its edge is

$$I' = \frac{1}{2}Mr^2 + Mr^2 = \frac{3}{2}Mr^2$$

Given $I = I'$ or $\frac{2}{5}MR^2 = \frac{3}{2}Mr^2$ or, $r^2 = \frac{4}{15}R^2$

or, $r = \frac{2R}{\sqrt{15}}$.

8. **(c) :** Here, resistance $R = 80\ \Omega$

and $X_L - X_C = 100 - 40 = 60\ \Omega$

$\therefore$ Impedance $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$= \sqrt{80^2 + 60^2} = 100$$

$\therefore$ Power factor, $\cos\phi = \frac{R}{Z} = \frac{80}{100} = 0.8$

9. **(b):** Since, $I = I_0(1 - e^{t/\tau})$ where τ = time constant

$\therefore \quad \frac{3}{4}I_0 = I_0(1 - e^{-t/\tau})$

$\Rightarrow \quad \frac{3}{4} = 1 - e^{-t/\tau} \quad \Rightarrow \quad e^{-t/\tau} = \frac{1}{4}$

or $\quad \frac{-t}{\tau}\ln e = \ln\frac{1}{4} \quad \Rightarrow \quad \frac{-4}{\tau} = -2\ln 2 \qquad [\because t = 4\text{ s}]$

$\Rightarrow \quad \tau = \frac{2}{\ln 2}$ s

10. **(b):** Linear momentum is conserved.

$\therefore \quad p_1 = p_2$

But $p = \sqrt{2mK}$ where K = kinetic energy.

and $\sqrt{2(216\,m)K_1} = \sqrt{2(4\,m)K_2}$

where K_2 is for α-particle and K_1 is for nucleus.

or $\quad 216K_1 = 4K_2 \quad$ or $K_2 = 54\,K_1 \qquad ...(i)$

Given p, $K_1 + K_2 = 5.5$ MeV $\qquad ...(ii)$

From (i) and (ii), $K_1 + 54\,K_1 = 5.5$ MeV

or $\quad 55\,K_1 = 5.5$ MeV or $K_1 = \frac{5.5}{55}$

$K_1 = \frac{1}{10}$ MeV

$\therefore \quad K_2 = 54K_1$ or $K_2 = \frac{54}{10}$ MeV

or $\quad K_2 = 5.4$ MeV

$\therefore \quad$ Kinetic energy of α-particle = 5.4 MeV.